National University of Ireland, Galway
Ollscoil na hÉireann, Gaillimh

College of Engineering and Informatics

Engineering Maths Qualifying Examination

PAST PAPERS

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENGINEERING MATHS QUALIFYING EXAMINATION 2018

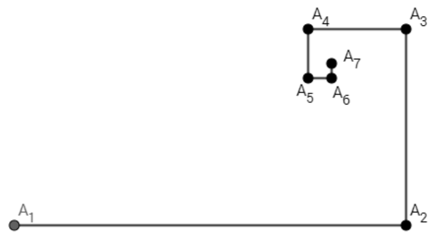
First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. **(a)** A point travels 8 units from A_1 to A_2, then turns left and travels 4 units upwards to A_3, as shown. At each stage, after the first one, the point moves left by half the distance of the previous stage. This pattern continues indefinitely.



(i) Find the length of the line segment $[A_{10}A_{11}]$.

(ii) Find the total length of all line segments, if the pattern continues indefinitely.

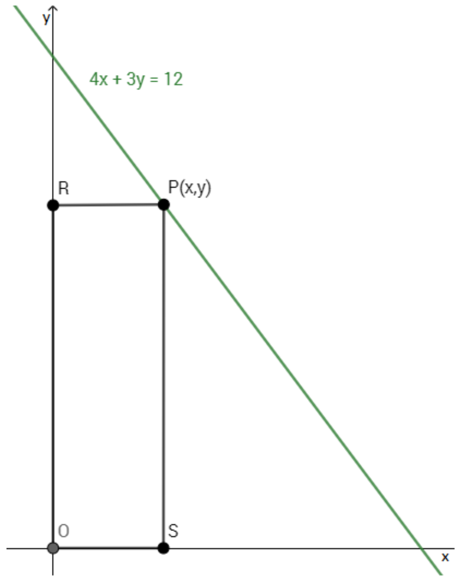
(iii) Determine the orientation (vertical or horizontal) of the line segment $[A_{2017}, A_{2018}]$.

(b) Find the sum of all natural numbers not exceeding 2018 which are divisible by 11.

(c) Prove that the number of diagonals in an n-sided polygon is $\dfrac{n(n-3)}{2}$, where $n \geq 3$.

2. **(a)** An alloy is a mixture of metals. One alloy is made up of 25% copper and 75% tin. Another alloy is made up of 80% copper and 20% tin. The two alloys are melted together and give a mixture of 50% copper and 50% tin. If the mixture weighs 330g, find the masses of the original alloys.

(b) The point $P(x, y)$ lies on the line $4x + 3y = 12$ as shown in the diagram, and $ORPS$ is a rectangle.



(i) Express the coordinates of P in terms of x only.

(ii) Write the area of the rectangle $ORPS$ in terms of x.

(iii) Find the maximum area of the rectangle $ORPS$.

3. (a) Differentiate the function $f(x) = 4x + \dfrac{4}{x}$ for $x \neq 0$. Hence find the coordinates of the minimum and maximum turning points.

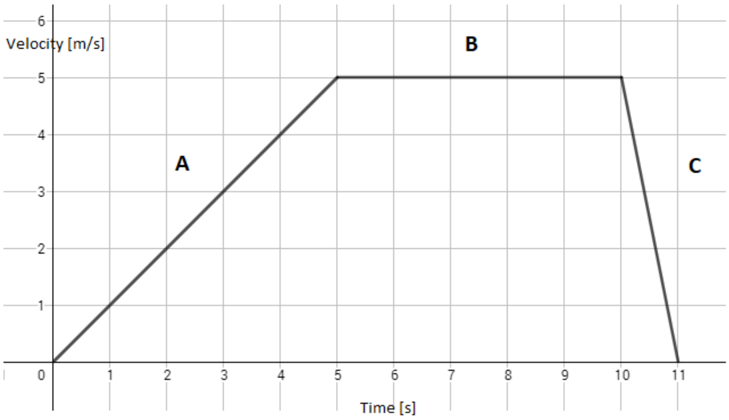
(b) Find the equation of the tangent to the curve $y = xe^x$ at the point $(0, 0)$.

(c) A cylindrical metal rod has a radius of r cm and length of $4r$ cm.

(i) Express the volume V of the rod in terms of r.

(ii) If the volume is increasing at the rate of 8 cm^3 per second, find the rate, in cm per second, at which the radius is increasing when the radius is 2 cm.

4. **(a)** A velocity-time graph is shown below.



(i) The slope of a line on a velocity-time graph represents the acceleration of the object. What is the acceleration in the sections A, B and C of the graph above?

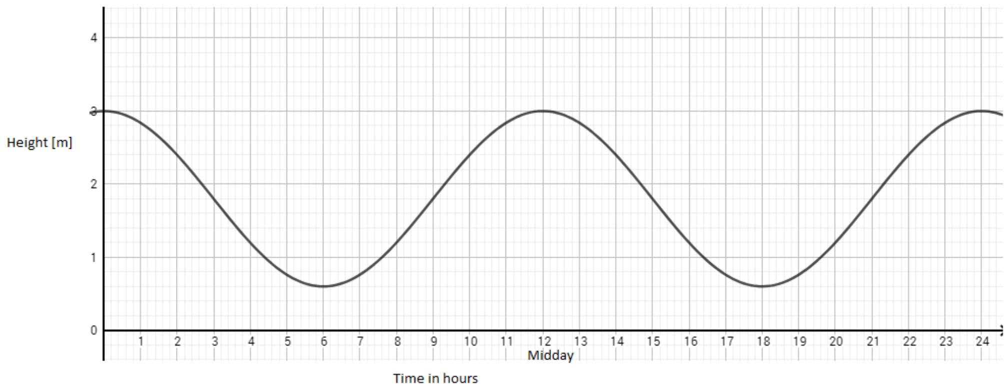
(ii) The area under this velocity-time graph represents the distance travelled. Find the distance covered by the object in the graph above.

(b) Find the following integrals.

(i) $\displaystyle\int \left(\sin x + e^{2x}\right) dx$

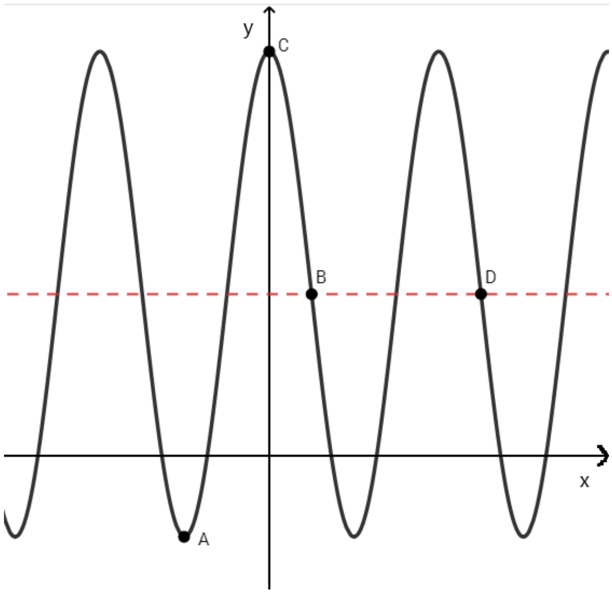
(ii) $\displaystyle\int_1^2 \left(x^2 + \frac{1}{x}\right) dx$

5. **(a)** The height of water in a harbour on a certain day changes between 0.6 metres at low tide and 3 metres at high tide, and can be modelled by a trigonometric function whose graph is shown below.



If a ship requires a minimum water level of 2 metres to safely navigate, use the graph to estimate the maximum amount of time that the ship can spend in the harbour, without resting on the sea-bed.

(b) The graph of a trigonometric function $y = f(x)$, where x is in degrees, is shown below. The coordinates of the points A and B are $(-30, -1)$ and $(15, 2)$ respectively, and B lies on the midway line.



(i) Find the coordinates of the points C and D.

(ii) Write the amplitude and the period of the function above.

(ii) Write the equation of the graph.

6. **(a)** Write the following in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

(i) $(2i^{14})^3$

(ii) $\dfrac{2+3i}{-1+3i}$

(b) Let z be the complex number given by $z = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$, where $i^2 = -1$.

(i) Calculate the product $\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)$ and write your answer in rectangular form $a + bi$, where $a, b \in \mathbb{R}$.

(ii) Find the modulus and argument of z.

(iii) Write z in polar form.

(iv) Using de Moivre's theorem, or otherwise, calculate z^4 to confirm that $z^4 = z$.

(v) Hence find the four complex numbers w such that $w^4 = z$ and illustrate them on an Argand diagram.

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENGINEERING MATHS QUALIFYING EXAMINATION 2018

Second Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. **(a)** A circle c passes through the points $A(0,0), B(6,0)$ and $C(6,8)$.

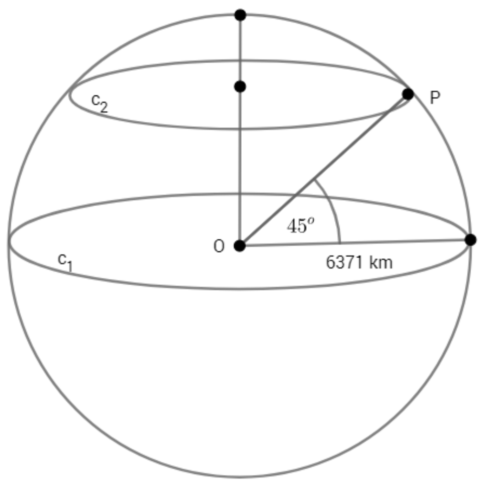
(i) Find the equation of the circle c. Write the length of the radius and the coordinates of the centre.

(ii) Find the measure of the angle BCA, correct to the nearest degree.

(iii) Find the equation of the tangent l to the circle at C.

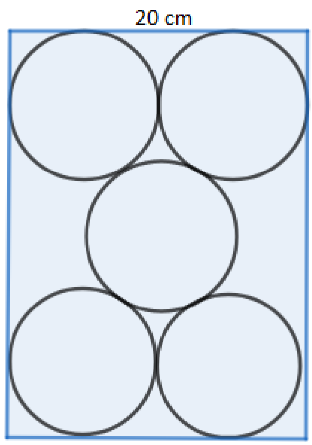
(iv) Find the coordinates of the points where l cuts the x-axis and the y-axis and hence, find the area of the triangle formed by l and the two axes.

(b) Assume the earth is a sphere with radius 6371 km. On the diagram, circle c_1 represents the equator and c_2 represents the circle that is at latitude 45°. Both circles are on parallel planes. The point O is the centre of the earth and the point P lies at latitude 45° north of the equator. Find the length (circumference) of c_2 to the nearest km.



2. **(a)** A ball is packed in a cube with the edge length of 10 cm and it touches all the faces of the cube. What percentage of the cube is unoccupied?

(b) Five identical balls fit exactly into a rectangular box of width 20 cm, as shown. Find the volume of the box.



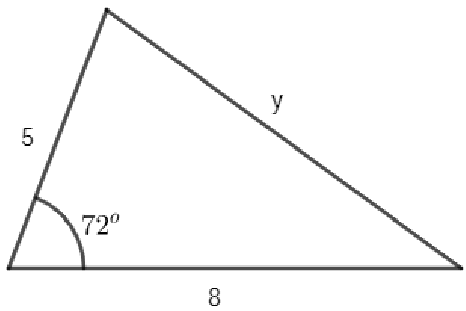
3. **(a)** Find the equation of the line l through the point $(-3, 4)$, which divides the line segment from $(-6, 2)$ to $(-3, -4)$ internally in the ratio $1 : 2$.

(b) Let $ABCD$ be a parallelogram whose vertices have coordinates $A(2, -1)$, $B(4, 0)$, $C(5, 3)$ and $D(3, 2)$.

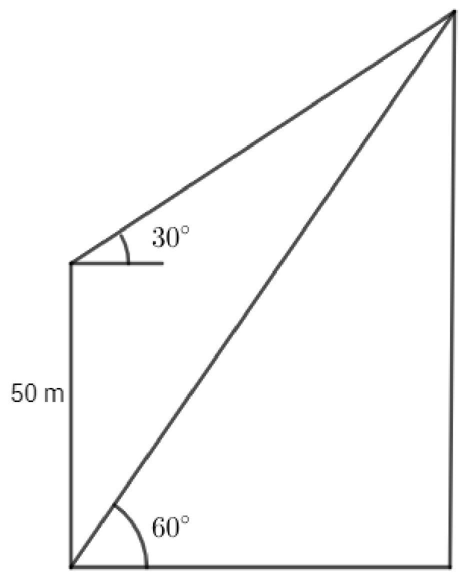
(i) Find the area of the parallelogram $ABCD$.

(ii) Let P be any point on the line segment $[CD]$. Prove that the area of the triangle ABP is half the area of the parallelogram.

4. **(a)** Find the missing length y in the diagram below, correct to two decimal places.



(b) A vertical 50 metres high tower and a vertical column are situated on horizontal ground. From the foot of the tower, the angle of elevation of the top of the column is 60°, while from the top of the tower, the angle of elevation is 30°. Find the height of the column.



5. (a) How many arrangements are there of the letters LEAVING:

(i) if there are no restrictions,

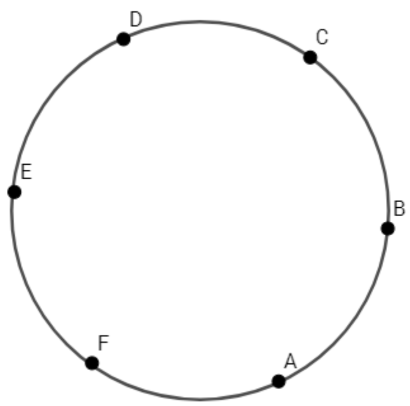
(ii) if AE are side by side, in that order,

(iii) if A and E are separated,

(iv) if consonants and vowels alternate?

If three letters are chosen at random from the word LEAVING without replacement, what are the chances that the first letter is N, second is I and third is L?

(b) How many triangles can be formed from six points regularly spaced on the circle, as shown in the diagram. How many of those triangles are isosceles?



6. **(a)** A coin is tossed five times.

(i) What is the probability of getting three heads and two tails?

(ii) Find the probability that the third head occurs on the fifth (last) toss.

(b) Suppose that in a certain diagnostic screening programme the probability of a misdiagnosis is $\frac{1}{30}$. In a sample of 100 cases independent of each other, what is the probability that 5 cases were misdiagnosed?

(c) A jury of 12 people is selected from a group of 10 men and 10 women. Find the probability that the jury selected has more women than men.

OLLSCOIL NA hÉIREANN, GAILLIMH

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ENTRANCE EXAMINATION 2017

MATHEMATICS

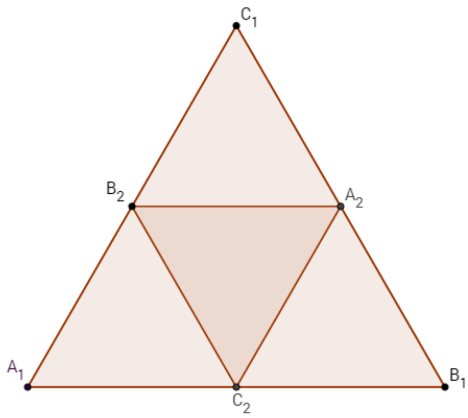
First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. (a) The length of each side in an equilateral triangle $A_1B_1C_1$ is 24 cm. The midpoints of each side are joined to form a new equilateral triangle $A_2B_2C_2$. This pattern continues indefinitely.



(i) Find the length of the perimeter of the fifth triangle in the sequence.

(ii) Find the length of the perimeter of the nth triangle in the sequence.

(b) How many terms of the arithmetic sequence $3, 7, 11, ...$ must be added to give a total of 990?

(c) Twenty water tanks are decreasing in size so that the volume of each tank is half the volume of the previous one. The first tank is empty, but the remaining 19 tanks are full of water. Explain if it is possible for the first tank to hold all of the water from the other 19 tanks.

2. **(a)** A restaurant bill that comes to €200 is to be divided equally among a group of people. However, three people leave before the bill is settled and the remainder of the group have to pay an extra €15 each. How many people were originally in the group?

(b) The rate of change of a population of bacteria can be modelled by the differential equation $\frac{dP}{dt} = kP$, where k is a constant and P is the population size at time t, t is measured in minutes.

(i) Show that $P = Ce^{kt}$ is a solution to the differential equation $\frac{dP}{dt} = kP$. (C is a constant.)

(ii) If the population has grown from its initial size of 1000 to 32000 in 60 minutes, find the value of k.

3. **(a)** A stone is thrown vertically upwards. The height h (in metres) of the stone after t seconds is given by

$$h = 1 + 10t - 5t^2.$$

Find:

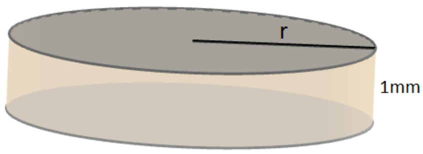
(i) the initial height,

(ii) the velocity when $t = 2$,

(iii) the maximum height reached by the stone.

(b) An oil spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^6 cm^3 per minute. The spilled oil forms a circular oil slick 1 millimetre thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.



(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

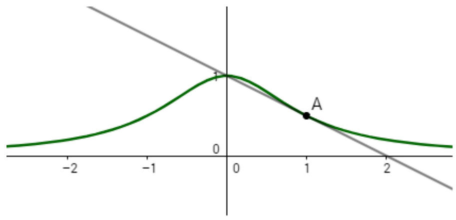
4. **(a)** Differentiate the following with respect to x.

(i) $(3x + 2)^2$

(ii) $e^{5x} + \sqrt{x}$

(iii) $e^x \cos(2x)$

(b) By differentiating y with respect to x, find an equation of the tangent to the curve $y = \dfrac{1}{x^2+1}$ at $A\left(1, \dfrac{1}{2}\right)$.



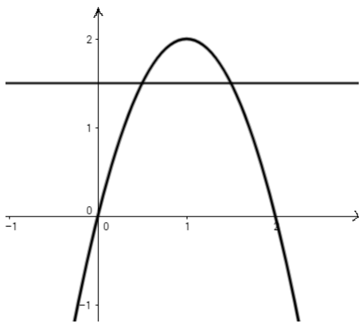
5. **(a)** Find the following indefinite integrals.

(i) $\int (x^2 - 7x + 2)dx$

(ii) $\int \cos(2x)dx$

(iii) $\int e^{4x}dx$

(b) Calculate the area of the region bounded by the curve $y = -2x^2 + 4x$ and the line $y = \dfrac{3}{2}$.



6. **(a)** Write the following in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

(i) $(1+i)(1+i)$

(ii) $\dfrac{3-2i}{1-3i}$

(b) Given is a complex number $z = -1 + \sqrt{3}i$.

(i) Find the modulus and argument of z.

(ii) Write z in polar form.

(iii) Using de Moivre's theorem, or otherwise, evaluate $(-1 + \sqrt{3}i)^{18}$.

(c) Solve the equation $z^2 + z + 1 = 0$, where $z \in \mathbb{C}$.

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ENTRANCE EXAMINATION 2017

MATHEMATICS

Second Paper

Time allowed: *Two* hours

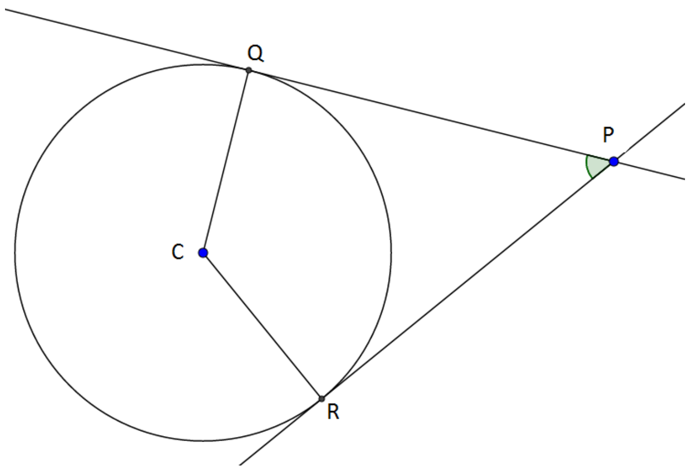
Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. Given is a circle s with equation

$$s: \; x^2 + y^2 + 4x + 2y - 12 = 0$$

and the point $P(7, 1)$ outside the circle.



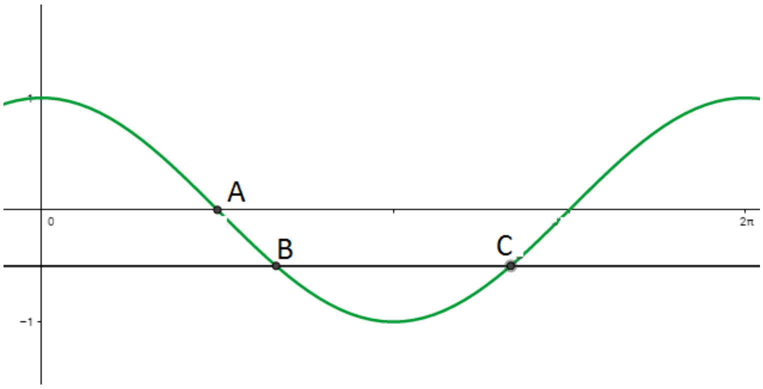
(a) Find the length of the radius and the coordinates of the centre C.

(b) Find the length of the tangent from the point P to the point Q on the circle s.

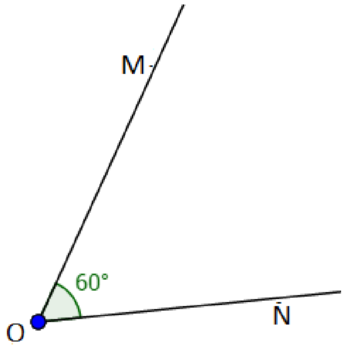
(c) Find the equations of the two tangents from P touching the circle at Q and R.

(d) Find $|\angle QPR|$, correct to the nearest degree.

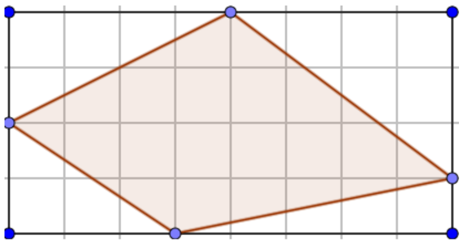
2. **(a)** The graph of $y = \cos(x)$ and the graph of $y = -0.5$ are shown below. Find the coordinates of A, B and C.



(b) Two cars M and N travel away from O as shown. Car M moves at a constant speed x km/h and car N moves at a constant speed y km/h. How far apart are they after t hours. [*Hint:* Cosine rule can be used.]



3. **(a)** The shaded area in the diagram below represents a lawn area in a rectangular garden. If the area of the garden is 320 m^2, find the area of the lawn.



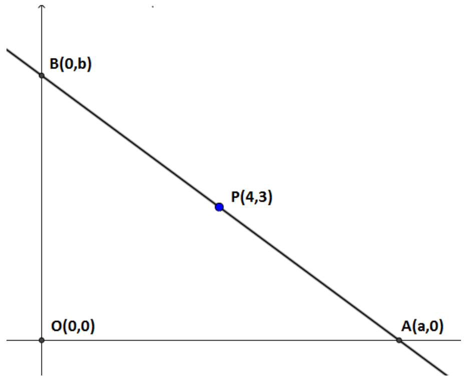
(b) Oil flows through a cylindrical pipe at a constant speed of 1.5 m/s. The internal diameter of the pipe is 1.2 metres.

i. The rate of flow of a liquid in a pipe is the volume of liquid which passes through the pipe per second. Calculate the rate of flow of oil in the pipe.

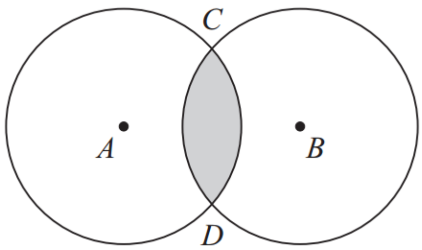
ii. The oil is collected in cylindrical tanks of base diameter 10 m and height 20 m. Find the time it takes to fill one such tank, correct to the nearest minute.

4. **(a)** Find the equation of the perpendicular bisector of the line segment $[AB]$, where $A(-7, 5)$ and $B(13, -11)$.

(b) The three points $A(a, 0)$, $B(0, b)$ and $O(0, 0)$ form a triangle of area 27 square units. The point $P(4, 3)$ is on the line AB. Find a and b.



5. **(a)** Two circles, each of radius 4 cm, intersect at the points C and D, as shown. The distance between the centres of the circles, A and B, is 6 cm.



(i) Find the measure of the angle CAD, correct to two decimal places.

(ii) Hence, find the area of the shaded region, correct to one decimal place.

(b) A cylindrical container, with base radius 9 cm and height 20 cm, is filled with water to a height of 14 cm. Find the least number of solid spheres of radius 3 cm that must be submerged in the water to cause the water to flow out of the top of the cylinder.

6. **(a)** A PIN code for a debit card consists of four digits.

(i) How many different codes are possible?

(ii) A thief has stolen a debit card. He does not know the complete code, but knows that there are at least two zeros in it. How many different possible codes fit this description?

(b) Each of the first six prime numbers is written on a separate card. The cards are shuffled and two cards are selected without replacement. What is the probability that the sum of the numbers selected is prime?

(c) Four letters are chosen at random from the word EXAMINATION without replacement. What are the chances that the first letter is E, second is X, third is I and fourth is T?

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2016

MATHEMATICS

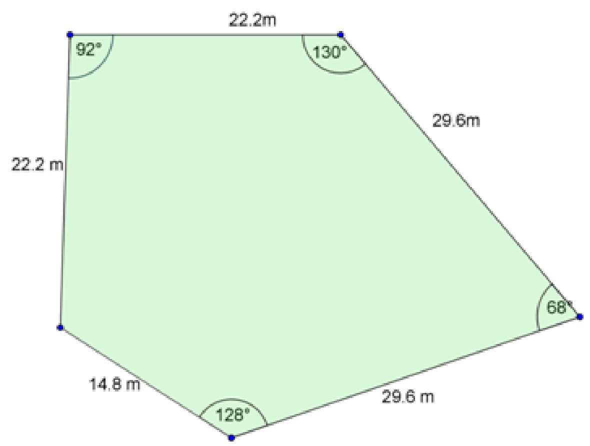
First Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. **(a)** A building site can be approximated by a pentagon as shown below.



(i) Find the area of the site.

(ii) A hectare is 10 000 m^2. What fraction of a hectare is this site?

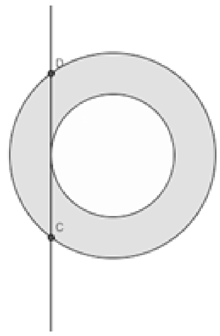
(b) The line $y = 0.5$ intersects the graph of the function $y = \sin x$ $(0 \leq x \leq \pi)$ at the points A and B. Find the length of the line segment $[AB]$.

2. **(a)** Find the values of a and b given that

$$\frac{4\cos^2\theta - 3}{1 - 2\sin\theta} = a + b\sin\theta$$

holds for all θ such that $\sin\theta \neq \frac{1}{2}$.

(b) The diagram shows a metal washer. The line segment $[DC]$ is 36 mm long.



(i) Find the area of the shaded region.

(ii) If the washer is 0.1 cm thick find the volume of metal in the washer.

(iii) If 1 cm^3 of the metal has mass 5g, find the mass of the washer.

(iv) If the material from which the washer is to be manufactured costs €250 per tonne, find the cost of the material required to manufacture 120 000 washers.

3. (a) Find the length of the tangent from the point $(-9, 3)$ to the circle

$$(x-2)^2 + (y-1)^2 = 25.$$

(b) Find the values of k for which $x + 4y + k = 0$ is a tangent to the circle $x^2 + y^2 - 2x + 2y - 15 = 0$.

(c) The circles $c_1 : (x+3)^2 + (y+2)^2 = 4$ and $c_2 : x^2 + y^2 - 2x - 2y - 7 = 0$ touch externally.

(i) Find the coordinates of the point of contact of c_1 and c_2.

(ii) Hence, or otherwise, find the equation of the tangent common to c_1 and c_2.

4. (a) Find all the positive values of x and y which satisfy the following system of equations:

$$x + xy + xy^2 = 26,$$

$$x^2y + x^2y^2 + x^2y^3 = 156.$$

(b) A shop assistant is arranging a triangular display of tins so as to have one tin in the top row, two in the second, three in the third row and so on. If there are 100 tins altogether, how many rows can be completed and how many tins will be left over?

(c) $T_1, T_2, T_3, ...$ is an arithmetic sequence with common difference 1. Given that $T_1 + T_2 + ... + T_{98} = 137$, find the value of $T_2 + T_4 + T_6 + ... + T_{98}$.

5. **(a)** There is a pole in a lake. One-half of the pole is in the ground, another one-third of it is covered by water, and 9 metres is out of the water. What is the total length of the pole in metres?

(b) Solve for x the following equation $\dfrac{2}{e^x} = e^x - 1$.

(c) Find all the solutions to the equation $\cos A + \sin A = \sqrt{\dfrac{3}{2}}$ in the domain $0 \leq A \leq \pi$.

6. **(a)** Determine $(1 + i)^8$ in its simplest form.

(b) The complex number z satisfies the equation $|z| = |z + 2|$.

(i) Show that the real part of z is -1.

(ii) That complex number z also satisfies $|z| = 2$. Find the two possible values of the imaginary part of z.

(iii) Show these two possible values of z on an Argand diagram and write them in polar form.

(c) Find the value of k such that the product $(2 + 3i)(2 + ki)$ is a real number.

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MATHEMATICS

Second Paper

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Candidates for Computer Science & Information Technology and Project & Construction Management should take **4** questions out of 6. All other candidates should take **5** questions out of 6.

Formulae and Tables booklet: REQUIRED

1. The lines $L_1 : 3x + 4y + 4 = 0$ and $L_2 : 5x + 12y + 28 = 0$ cut the x-axis at points A and B respectively.

(a) Find the coordinates of A and B.

(b) The lines L_1 and L_2 intersect at P. Find the coordinates of P.

(c) Let $K(k, 0)$ be a point on the line segment $[AB]$ which is equidistant from L_1 and L_2. Find (i) the coordinates of K and (ii) its distance from L_1 and L_2.

(d) Hence, or otherwise, find the equation of the line bisecting the acute angle between L_1 and L_2.

2. **(a)** An access code to a computer system consists of two different letters followed by three different digits.

(i) How many different codes are possible?

(ii) A user has been given a code, but cannot remember it fully. All that the person can remember is that the first letter is C and the first digit is 0. How many different possible codes fit this description?

(b) Each of the first six prime numbers is written on a separate card. The cards are shuffled and two cards are selected without replacement. What is the probability that the sum of the numbers selected is prime?

(c) A box contains three coins: one coin is fair, one coin is two-headed, and one coin is weighted so that the probability of heads appearing is $\frac{1}{3}$. A coin is selected at random and tossed. Find the probability that a head will appear.

3. (a) The displacement, x cm, of the end of a spring at time t seconds is given by

$$x = e^{-2t} \sin(20\pi t).$$

Find the velocity of that end of the spring after 2 seconds.

(b) On a building site, sand is stored in a container which is 4 metres above ground. The sand is released through an opening in the floor of the container and forms a conical mound in which the height is equal to the diameter of its base.

(i) If the sand is released at the rate of 500π cm^3 per second, show that it will take less than 3 hours for the top of the conical mound of sand to reach the container.

(ii) Find the rate at which the height of the mound is increasing when the height is 2 metres.

4. The line AC is a tangent to the circle $(x-7)^2 + (y-1)^2 = 20$ at the point $A(3,3)$. AC is also a tangent to the curve $f(x) = x^2 + 2x - 3$ at the point C.

(a) Find the slope of the line AC.

(b) Find the equation of AC.

(c) Use two different methods to find the coordinates of the point C.

(d) Show that the area of the circle is more than four times greater than the area of the triangle ABC, where B is the centre of the circle.

5. (a) The graph of the function $f(x) = ax^2 + bx + c$ is a curve which passes through the points $A(0,-2)$ and $B(2,2)$. The minimum point of the curve is at $x = 0.5$.

(i) Find the values of a, b and c.

(ii) Find the average rate of change of $f(x)$ between A and B.

(iii) Find the point on the curve where the instantaneous rate of change is equal to the rate in **(ii)**, above.

(b) The point O is the intersection of two roads that cross at right angles. One car is 80 metres north of O and travels towards O at 20 m/s. A second car is 100 metres west of O and travels due east towards O also at 20 m/s.

(i) Explain why after t seconds their distance apart, d, is given by

$$d = \sqrt{(100-20t)^2 + (80-20t)^2}.$$

(ii) Show that the minimum distance between the two cars is $10\sqrt{2}$ metres.

6. (a) Find:

(i) $\displaystyle\int \left(\frac{1}{x^2} + x^2\right) dx,$ **(ii)** $\displaystyle\int_0^{\frac{\pi}{2}} \cos(2x)\, dx.$

(b) The area between the curve $y = x^2 + 4x$ and the x-axis, for $0 \leq x \leq 3$, is equal to the area of a rectangle of base 3 and height h. Find the value of h.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2015

MATHEMATICS

First Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

Log Tables: REQUIRED

1. (a) The coordinates of three vertices of a parallelogram $ABCD$ are:

$$A\,(-5,\,1),\ \ B\,(2,\,4),\ C\,(1,\,1).$$

calculate the area of the parallelogram

(b) A line divides a circle of radius length 2 into two unequal arcs. Given that the length of the smaller arc is $\frac{\pi}{3}$, determine the area of the region bounded by the smaller arc and the line.

(c) Determine all the values of θ, where $0 < \theta \leq 2\pi$, such that $\sin^2\theta = 1 - \cos\theta$.

2. (a) For $x > \dfrac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5\left(\frac{3}{2x-1}\right).$$

(b) Find the sum of all the natural numbers less than 2015 which are divisible by 7.

(c) The volume of a rectangular solid is 8cm^3, and its total surface area is 32cm^2. The three dimensions (length, width and height) are in geometric progression (a, ar, ar^2). Determine the sum of the length of all the 12 edges of the solid (in cm).

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5, 5)$, $(4, 8)$, $(-7, -3)$, the co–ordinates of its centre and the length of the radius.

(b) Find the equation of the circle which touches the y–axis at the point (0, 3) and passes through the point (1, 0). Hence find the co–ordinates of its centre and the length of its radius.

(c) Find the radius and centre of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $3x + 4y = 4$.

4. (a) If $(x,\, y)$ is a solution to the system of equations (i), (ii) below

$$\begin{aligned} xy &= 8 \qquad (i) \\ x^2y + xy^2 + x + y &= 63 \qquad (ii) \end{aligned}$$

determine the value of $x^2 + y^2$.

(b) There are four points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these eight points as vertices?

5. (a) Writing $\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$ in modulus-argument form (or otherwise), determine $\left(\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}\right)^{11}$ (where $i = \sqrt{-1}$).

(b) Determine the complex number z such that $\dfrac{2z - 3i}{z + 2} = -5 + i$ (where $i = \sqrt{-1}$).

(c) Show that the complex numbers z $(= x + iy)$ which satisfy

$$|z - 1 - i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

6. (a) Consider the function $f(x) := \ln(1 + x^2)$. Calculate the first derivative $f'(x)$, also written $\dfrac{df}{dx}$. Calculate the second derivative $f''(x)$, also written $\dfrac{d^2 f}{dx^2}$.
Hence or otherwise show that $f(x)$ has a minimum at $x = 0$ and two points of inflection at $x = \pm 1$.

(b) Find the rational numbers a, b such that

$$f(x) = \frac{1}{(x + 5)(x + 7)} = \frac{a}{x + 5} + \frac{b}{x + 7}.$$

Let x take the values $0, 1, 2, ..., n$.
Compute $f(0) + f(1) + ... + f(n)$. Deduce that the area between the curve $y = f(x)$ and the positive x-axis is at most $\dfrac{11}{60}$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2015

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

Log Tables: REQUIRED

1. (a) Find the equations of the lines represented by the equation

$$2x^2 + 5xy - 3y^2 + 7x + 14y + 5 = 0.$$

(b) A line containing the point P with coordinates $(5, 6)$ touches the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ at the point Q. Calculate the distance PQ.

(c) The lines $L_1 : 3x + 4y + 4 = 0$ and $L_2 : 5x + 12y + 28 = 0$ cut the x-axis at points A and B respectively. Let K be a point in the segment $[AB]$ which is equidistant from L_1 and L_2. Find (i) the coordinates of K and (ii) this distance from L_1 and L_2.

2. Let A, B, C be the three angles in a triangle and let a, b, c be the lengths of the sides opposite the respective angles A, B, C. Let r denote the radius of the circle inscribed in the triangle (and tangent to the sides of the triangle).
PROVE:

(a) $\Delta = sr$ where Δ is the area of the triangle and $s = \dfrac{a+b+c}{2}$, the so–called semi–perimeter of the triangle.

(b) $r\left(\cot\dfrac{B}{2} + \cot\dfrac{C}{2}\right) = a.$

(c) $r\cos\dfrac{A}{2} = a\left(\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right).$

p.t.o.

3. (a) How many different 15-letter "words" can be formed from the letters of the word **UNPREPOSSESSING**? Explain your answer. [Note that interchanging any of the four letters S among themselves does not give a different "word".]
How many of these "words" have all four **S**'s together?

(b) If repetitions are **NOT** allowed

(i) how many 3–digit numbers can be formed from the set $\{3, 4, 5, 6, 7, 8, 9\}$?

(ii) How many of these are **ODD** numbers?

(iii) How many are greater than 600?

(iv) How many are divisible by 4?

4. (a) Show that $y = Ae^x + Be^{2x}$, where A and B are constants, is a solution of the equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

Given that $y = 1$ when $x = 0$, and $\frac{dy}{dx} = 0$ when $x = 0$, determine the value of the constants A and B.

(b) Consider the curve

$$y = \frac{2x}{x+1}$$

for $x \geq 0$. By differentiating y with respect to x, or otherwise, find the equation of the tangent to the curve at $x = 1$.

5. (a) Differentiate with respect to x:

(i) $(\sin x)^2$ (ii) $\sqrt{\frac{x+1}{x+2}}$; (iii) $2x \ln x$.

(b) Find the maxima and minima of the function

$$f(x) = x^3 - x^2 - x + 2.$$

and sketch its graph on the interval $(-2, 2)$ (do **NOT** use graph paper). How many real roots does the function $f(x)$ have?

6. (a) Evaluate the following integrals:

$$\int \sin 4x \; dx; \qquad \int_0^1 2x\sqrt{x^2+1} \; dx; \qquad \int_0^{\pi/2} \sin^3 x \; dx.$$

(b) Sketch the graphs of the functions $y = x^2$ and $y = 2 - x$ (do **NOT** use graph paper) and find the area between their graphs.

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COLLEGE OF ENGINEERING AND INFORMATICS

ENTRANCE EXAMINATION 2014

MATHEMATICS

First Paper

Time allowed: 2 hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

DO NOT USE GRAPH PAPER. LOG BOOKS WILL BE PROVIDED BY THE EXAMS OFFICE.
CALCULATORS MAY BE USED.

1. (a) Consider a triangle in which two of its angles A, B are acute. Given that $\tan(A) = \dfrac{3}{4}$ and that $\sin(B) = \dfrac{5}{13}$, express the cosine of the third angle , $\cos(C)$, as a fraction $\dfrac{p}{q}$, where p, and q are integers.

(b) If $\sin\theta = \dfrac{1-x}{1+x}$ $\quad (x \geq 0)$, $\quad$ express $\cos\theta$ in terms of x and hence show that

$$\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{x}.$$

(c) Determine all the values of θ, where $0 < \theta < \pi$ such that $6\cos^2(\theta) - \sin(\theta) = 4$.

2. (a) Find the smallest positive integer a, such that 2014 is a term of the following arithmetic sequence:

$$a,\ a+7,\ a+14,\ a+21, \ldots$$

(b) Find the sum of all the natural numbers less than 2014 which are divisible by 11.

(c) The volume of a rectangular solid is 8cm^3, and its total surface area is 32cm^2. The three dimensions (length, width and height) are in geometric progression $(a,\ ar,\ ar^2)$. Determine the sum of the length of all the 12 edges of the solid (in cm).

p.t.o.

3. (a) Find the distance from the point (5, 8) to the nearest point on the circle $2x^2 + 2y^2 + 4x - 8y = 6$.

(b) The x–axis is a tangent to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Prove that $g^2 = c$.

(c) Three straight lines l, m, n have slope $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. All three lines have the same y–intercept. Given that the sum of the x–intercepts of the three lines is 36, calculate the value of the y–intercept.

4. (a) If (x, y) is a solution to the system of equations (i), (ii) below

$$\begin{aligned} xy &= 6 \qquad (i) \\ x^2y + xy^2 + x + y &= 63 \qquad (ii) \end{aligned}$$

show that $x^2 + y^2 = 69$.

(b) There are four points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these eight points as vertices?

5. (a) Write the complex number $w = -8 + 8\sqrt{3}i$ in modulus–argument form, ie in the form $r(\cos\theta + i\sin\theta)$ (where $i = \sqrt{-1}$).

(b) Using De Moivre's Theorem or otherwise, show that $z = \sqrt{3} + i$ satisfies $z^4 = -8 + 8\sqrt{3}i$.

(c) Show that the complex numbers z $(= x + iy)$ which satisfy

$$|z - 1 - 2i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

6. (a) Consider the function $f(x) := \ln(1 + x^2)$. Calculate the first derivative $f'(x)$, also written $\frac{df}{dx}$.

Calculate the second derivative $f''(x)$, also written $\frac{d^2f}{dx^2}$.

Hence or otherwise show that $f(x)$ has a minimum at $x = 0$ and two points of inflection at $x = \pm 1$.

(b) Let a, b be **positive** real numbers. Set $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

(i) Show that $G = \sqrt{AH}$.

(ii) **Prove** that $A - G \geq 0$ with equality if and only if $a = b$.

(iii) Deduce that $A > G > H$ unless $a = b$ in which case show that $A = G = H$.

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COLLEGE OF ENGINEERING AND INFORMATICS

ENTRANCE EXAMINATION 2014

MATHEMATICS

Second Paper

Time allowed: 2 hours

Candidates for Computer Science & Information Technology and Project & Construction Management should do 4 questions. All other candidates should do 5 questions.

DO NOT USE GRAPH PAPER. LOG BOOKS WILL BE PROVIDED BY THE EXAMS OFFICE.
CALCULATORS MAY BE USED.

1. The coordinates of three points A, B and C are: $A(2, 2)$, $B(6, -6)$, $C(-2, -3)$.

(a) Find the equation of the line AB.

(b) The line AB intersects the y-axis at D. Find the coordinates of D.

(c) Find the perpendicular distance from C to the line AB.

(d) Hence, find the area of the triangle ADC.

2. (a) The letters of the word HOLIDAYS are all arranged at random.

(i) How many arrangements are possible?

(ii) How many of these arrangements contain the letter group OLD in that order?

(b) A man is dealt five cards from an ordinary pack of 52 playing cards.

(i) In how many ways will the cards contain exactly two kings?

(ii) In how many ways will the cards contain at least one king?

(iii) In how many ways will the cards contain three of one kind and two of another kind, for example three jacks and two aces?

3. **(a)** The centre of a circle lies on the line $x - 2y - 1 = 0$. The x-axis and the line $y = 6$ are tangents to the circle. Find the equation of this circle.

(b) A different circle has equation $x^2 + y^2 - 6x - 12y + 41 = 0$.

(i) Show that this circle and the circle in part **(a)** touch externally.

(ii) Find the coordinates of the point where the two circles touch.

4. The function f is defined as $f(x) = x^3 + 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.

(a) Find the coordinates of the point where the graph of f cuts the y-axis.

(b) Find the coordinates of the points where the fraph of f cuts the x-axis.

(c) Find the coordinates of the local maximum turning point and of the local minimum turning point of f.

(d) Hence, sketch the graph of the function f for $-5 \leq x \leq 3$. Use an appropriate scale on each axis.

5. **(a)** Let $g(x) = -\sin(4x)$, where $x \in \mathbb{R}$, and let $h(x) = \dfrac{0.5x}{x+2}$, where $x \neq -2$, $x \in \mathbb{R}$.

(i) Find $g'(x)$, the derivative of $g(x)$, and $h'(x)$, the derivative of $h(x)$ at $x = 0$.

(ii) Use the derivatives obtained in **(i)** to determine the measure of the angle between the tangents to the curves $g(x)$ and $h(x)$ at the point $(0, 1)$.

(b) A small rocket is fired into the air from a fixed position on the ground. Its flight lasts ten seconds. The height, in metres, of the rocket above the ground after t seconds is given by

$$h = 10t - t^2.$$

(i) Find the times when the rocket will be at the height of 9 metres above the ground.

(ii) Find the maximum height reached by the rocket.

(iii) Find the speed of the rocket after 3 seconds.

6. **(a)** Find:

(i) $\displaystyle\int \left(\frac{1}{x^2} + x^2\right) dx$, **(ii)** $\displaystyle\int_0^{\frac{\pi}{4}} \cos(2x)\, dx$, **(iii)** $\displaystyle\int_0^1 (3x^2 + e^x)\, dx$.

(b) Find the area of the region bounded by the curve $y = x^2 - 4$ and the line $y = x + 2$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2013

MATHEMATICS

First Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

DO NOT USE GRAPH PAPER

1. (a) Show that

$$\frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

(b) If $\sin\theta = \dfrac{1-x}{1+x} \quad (x \geq 0),$ express $\cos\theta$ in terms of x and hence show that

$$\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{x}.$$

(c) Simplify the equation

$$\cos\left(y - \frac{\pi}{6}\right) - 3\,\sin\left(y + \frac{\pi}{3}\right) = 1$$

and hence (or otherwise) find one value of y which satisfies it.

2. (a) Find the sum of all the natural numbers less than 2013 which are divisible by 13.

(b) The numbers

$$5 + x, \quad 5x + 1, \quad 3 + 3x$$

are the first three terms in an **arithmetic** progression. Find the value of x and the sum of the first n terms of the corresponding series.

(c) Let $h(x) := 3x^2 - x$. Show that the sum of the twenty numbers $h(1)$, $h(2)$, $h(2^2), \ldots, h(2^{19})$ is equal to

$$2^{20}(2^{20} - 1).$$

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5,5)$, $(4,8)$, $(-7,-3)$, the co–ordinates of its centre and the length of the radius.

(b) The x–axis is a tangent to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Prove that $g^2 = c$.

(c) Find the radius and centre of each of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $x + y = 2$.

4. (a) Solve the set of simultaneous equations

$$\begin{array}{rcrcrcr} 4x & + & y & - & 2z & = & 3 \\ 2x & + & 2y & - & 2z & = & -4 \\ 2x & + & y & - & z & = & -3 \end{array}$$

(b) There are four points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these eight points as vertices?

5. (a) Write the complex number $w = -8 + 8\sqrt{3}i$ in modulus–argument form, ie in the form $r(\cos\theta + i\sin\theta)$ (where $i = \sqrt{-1}$).

(b) Using De Moivre's Theorem or otherwise, show that $z = \sqrt{3} + i$ satisfies $z^4 = 8 + 8\sqrt{3}i$.

(c) Show that the complex numbers z $(= x + iy)$ which satisfy

$$|z - 1 - 2i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

6. (a) For $x > \dfrac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5\left(\frac{3}{2x-1}\right).$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x+3}{x-4} < -2.$$

(c) Let a, b be **positive** real numbers. Set $A = \dfrac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \dfrac{2ab}{a+b}$.

(i) Show that $G = \sqrt{AH}$.

(ii) **Prove** that $A - G \geq 0$ with equality if and only if $a = b$.

(iii) Deduce that $A > G > H$ unless $a = b$ in which case show that $A = G = H$.

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ENTRANCE EXAMINATION 2013

MATHEMATICS

Second Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) A and B are the points $(-1, 4)$ and $(3, 7)$ respectively. Find the co-ordinates of the point that divides the line segment AB internally in the ratio $3 : 1$.

(b) (i) Find the equations of the lines l_1 and l_2 through the point P with co-ordinates $(3, 4)$ which make an angle of $45°$ with the line l_3 having the equation

$$x - 2y + 2 = 0.$$

(ii) Find the area of the triangle enclosed by the three lines l_1, l_2, l_3 in part (i).

2. (a) You must choose a new 4 digit ATM pin number from the numbers 5, 0, 3, 8. How many pin numbers in total are possible? What are the chances that your pin number is odd?

(b) In a train there are 8 seats, with 4 facing the front and 4 facing backwards. If 5 people sit down in the carriage, how many different ways can they be seated?

(i) If two of the people don't like sitting backwards, in how many ways can they be arranged?

(ii) Find the probability that two particular people will sit opposite each other.

3. (a) Write out the binomial expansion of $\left(x+\frac{1}{x}\right)^4$ and $\left(x-\frac{1}{x}\right)^4$. Hence, or otherwise, find the values of p, q and r in the following equation

$$\left(x+\frac{1}{x}\right)^4+\left(x-\frac{1}{x}\right)^4=px^4+q+\frac{r}{x^4}.$$

(b) A city council consists of 6 Liberal and 5 Labour councillors, from whom a committee of 5 is chosen. What is the probability that the Liberals have a majority?

4. (a) Differentiate with respect to x:

(i) $(2x^2+3x+1)(x^3-x+2)$, (ii) $\dfrac{5x}{x+4}$, (iii) $\cos(3x^2)$.

(b) Let $f(x)=x^2+3x-1$, where $x \in \mathbb{R}$. Consider the line l_1 which passes through the point $(2,0)$ and is a tangent to the curve $f(x)$ at the point $(-1,-3)$.

(i) Find the slope of l_1 using a slope formula.

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) Verify your answer to (i) by finding the value of $f'(x)$ at $x=-1$.

(iv) The line l_2 is perpendicular to l_1 and is also a tangent to the curve $f(x)$. Find the co-ordinates of the point at which l_2 touches the curve.

5. (a) Build a rectangular pen with three parallel partitions using 150 metres of fencing. What dimensions will maximise the total area of the pen?

(b) A stone is thrown vertically upwards. The height s metres, of the stone after t seconds is given by:

$$s=5(4t-t^2).$$

(i) Find the height of the stone after 1 second.

(ii) Show that the stone momentarily stops two seconds after being thrown, and find its height at that time.

(iii) Show that the acceleration of the stone is constant.

6. (a) Evaluate the following integrals:

(i) $\displaystyle\int_0^2 12e^{3x}\,dx$, (ii) $\displaystyle\int \sin(3x)\,dx$, (iii) $\displaystyle\int_0^2 (4x-x^3)\,dx$.

(b) Draw the graphs of $y=4x$ and $y=x^3$ in the domain $-2 \le x \le 2$, $x \in \mathbb{R}$. Find the area of the region enclosed by the two graphs.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2012

MATHEMATICS

First Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

DO NOT USE GRAPH PAPER

1. (a) Let α and β be the two roots of the equation

$$x^2 - 6x + 2 = 0.$$

Determine the value of $\alpha^3 + \beta^3$.

(b) Let α and β and γ be the roots of

$$x^3 + x^2 + x + 1 = 0.$$

Prove that

$$\alpha^{2012} + \beta^{2012} + \gamma^{2012} = 3.$$

2. (a) Find the sum of all the natural numbers less than 2012 which are divisible by 11.

(b) The numbers

$$3 + x, \quad 5x - 1, \quad 1 + 3x$$

are the first three terms in an **arithmetic** progression. Find the value of x and the sum of the first n terms of the corresponding series.

(c) Let $h(x) := 3x^2 - x$. Show that the sum of the twenty numbers $h(1)$, $h(2)$, $h(2^2), \ldots, h(2^{19})$ is equal to

$$2^{20}(2^{20} - 1).$$

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5, 5)$, $(4, 8)$, $(-7, -3)$, the co–ordinates of its centre and the length of the radius.

(b) The x–axis is a tangent to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Prove that $g^2 = c$.

(c) Find the radius and centre of each of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $x + y = 2$.

4. (a) For $x > \frac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5 \left(\frac{3}{2x - 1}\right).$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x + 3}{x - 4} < -2.$$

(c) There are three distinct points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these seven points as vertices?

5. (a) Show that

$$\frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha + i\sin\alpha - 1} = -i\cot\frac{\alpha}{2}$$

(where $i = \sqrt{-1}$).

(b) Show that the complex numbers z $(= x + iy)$ which satisfy

$$|z - 1 - 2i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

(c) Find also the set of complex numbers z which satisfy

$$|z - 1 - 2i| = |z + 1 - 2i|.$$

6. (a) Determine k such that

$$(1 + kn + 2n^2)(1 - kn + 2n^2) = 1 + 4n^4.$$

Hence or otherwise find real numbers a and b such that

$$\frac{4n}{1 + 4n^4} = \frac{a}{1 + kn + 2n^2} + \frac{b}{1 - kn + 2n^2}.$$

(b) Using part (a) or otherwise, calculate

$$s_n := \sum_{k=1}^{n} \frac{4k}{1 + 4k^4} \ .$$

By finding $\lim_{n \to \infty} s_n$, deduce that

$$\sum_{k=1}^{\infty} \frac{4k}{1 + 4k^4} = 1 \, .$$

(c) Find the **least** value of r such that

$$\sum_{k=1}^{r} \frac{4k}{1 + 4k^4} > \frac{999}{1000}$$

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2012

MATHEMATICS

Second Paper

Time allowed: *Two* hours

Candidates for Computer Science & Information Technology and Project & Construction Management should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) The line $L_1 : x - y + 3 = 0$ and $L_2 : 5x + y + 3 = 0$ intersect at the point P. Find an equation of the line through P perpendicular to L_1.

(b) The equations of two circles are:

$$\begin{aligned} c_1 : &\quad (x-3)^2 + (y-5)^2 = 5 \\ c_2 : &\quad x^2 + y^2 - 2x - 2y - 43 = 0 \end{aligned}$$

(i) Write down the centre and radius-length of each circle.
(ii) Prove that the circles are touching.
(iii) Verify that (4, 7) is the point that they have in common.

2. (a) A password for a website consists of capital letters A, B, C, ... Z and/or digits 0,1,2,... 9. The password has four such characters and starts with a letter. For example, BA7A, C999 and DGKK are allowed, but 7DCA is not.

Show that there are more than a million possible passwords.

(b) Six yachts compete in a race. All yachts finish the race and no two yachts finish the race at the same time.

(i) In how many different orders can the six yachts finish the race?
(ii) A person is asked to predict the correct order of the first two yachts to finish the race. How many different such predictions can be made?
(iii) A person selects two of the six yachts at random. What is the probability that the selected yachts are the first two yachts to finish the race?

3. (a) Consider the binomial expansion of $\left(x + \frac{1}{x}\right)^{10}$ for $x \neq 0$. Find the coefficient of x^8.

(b) Prove that if n is a positive integer, then

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$

(c) In a café there are 11 seats in a row at the counter. Six people are seated at random at the counter.
How much more likely is it that all six are seated together than that no two of them are seated together?

4. (a) Differentiate with respect to x:

(i) $(4x^2 - 5)^3$, (ii) $\dfrac{2 + x^2}{2 - x}$, (iii) $\sin(3x^2)$.

(b) Let $h(x) = 1 + 3x - x^2$, where $x \in \mathbb{R}$.

(i) Find the co-ordinates of the point P at which the curve $y = h(x)$ cuts the y-axis.

(ii) Find the equation of the tangent to the curve $y = h(x)$ at P.

(iii) The tangent to the curve $y = h(x)$ at $x = t$ is perpendicular to the tangent at P. Find the value of t.

5. (a) What is the largest possible area for a right triangle whose hypotenuse is 5 cm long? What are the other angles of this triangle? Justify your answer.

(b) A ball is thrown vertically down from the top of a high building. The distance the ball falls is given by

$$s = 3t + 5t^2,$$

where s is in metres, and t is the time in seconds from the instant the ball is thrown.

(i) Find the speed of the ball after 3 seconds.

(ii) Find the time t when the ball is falling at a speed of 23 m/s.

(iii) The ball hits the ground at a speed of 43 m/s. How high is the building?

6. (a) The function f is given by the formula $f(x) = x^2 + 1$. The tangents to the curve $y = f(x)$ at the points $(1, 2)$ and $(-1, 2)$ pass through the origin. Find the area of the region enclosed by these two tangents and the curve.

(b) Evaluate the following integrals:

(i) $\displaystyle\int x \cos(4x)\, dx$, (ii) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{1}{x^2} - \sin(3x)\, dx$, (iii) $\displaystyle\int_1^2 \frac{12}{3x - 2}\, dx$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2011

MATHEMATICS

First Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

DO NOT USE GRAPH PAPER

1. (a) Let α, β and γ be the three roots of the equation

$$x^3 - 3x^2 + x + 2 = 0.$$

Determine the value of $\alpha^3 + \beta^3 + \gamma^3$.

(b) Find the roots α, β, γ of the equation

$$3x^3 - 26x^2 + 52x - 24 = 0.$$

given that the roots are in geometric progression.

2. (a) Find the sum of all the natural numbers less than 2011 which are divisible by 7.

(b) The numbers

$$3 + x, \quad 5x - 1, \quad 1 + 3x$$

are the first three terms in an **arithmetic** progression. Find the value of x and the sum of the first n terms of the corresponding series.

(c) Show that

$$\frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}.$$

Hence, or othewise, determine

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}.$$

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5,5)$, $(4,8)$, $(-7,-3)$, the co–ordinates of its centre and the length of the radius.

(b) Find the equation of the circle which touches the y–axis at the point (0, 3) and passes through the point (1, 0). Hence find the co–ordinates of its centre and the length of its radius.

(c) Find the radius and centre of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $3x + 4y = 4$.

4. (a) Solve the simultaneous equations:

$$\begin{aligned} 2x - 3y &= 1 \\ x^2 + xy - 4y^2 &= 2 \end{aligned}$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x+3}{x-4} < -2.$$

(c) There are three distinct points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these seven points as vertices?

5. (a) Show that

$$\frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha + i\sin\alpha - 1} = -i\cot\frac{\alpha}{2}$$

(where $i = \sqrt{-1}$).

(b) Show that the complex numbers $z \ (= x + iy)$ which satisfy

$$|z - 1 - 2i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

(c) Find also the set of complex numbers z which satisfy

$$|z - 1 - 2i| = |z + 1 - 2i|.$$

6. Find the rational numbers a, b such that

$$f(x) = \frac{1}{(2x+3)(2x+5)} = \frac{a}{2x+3} + \frac{b}{2x+5}.$$

Let x take the values $0, 1, 2, ..., n$.
Compute $f(0) + f(1) + ... + f(n)$. Deduce that the area between the curve $y = f(x)$ and the positive x-axis is at most $\frac{1}{6}$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2011

MATHEMATICS

Second Paper

Time allowed: *Two* hours

Instructions for Candidates

Candidates for Computer Science & Information Technology and Project & Construction Management should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) The line $L_1 : 3x - 2y + 7 = 0$ and $L_2 : 5x + y + 3 = 0$ intersect at the point P. Find the equation of the line through P perpendicular to L_2.

(b) (i) Prove that

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}.$$

(ii) L is the line $tx + (t-2)y + 4 = 0$ where $t \in \mathbb{R}$. Given that the angle between L and the line $x - 3y + 1 = 0$ is $45°$, find all the possible values of t.

(c) A circle has its centre in the first quadrant. The x-axis is a tangent to the circle at the point $(3, 0)$. The circle cuts the y-axis at points that are 8 units apart. Find the equation of the circle.

p.t.o.

2. (a) The password for a mobile phone consists of five digits.

(i) How many passwords are possible?

(ii) How many of these passwords start with a 2 and finish with an odd digit?

(b) Seven horses run in a race. All horses finish the race and no two horses finish the race at the same time.

(i) In how many different orders can the seven horses finish the race?

(ii) A person is asked to predict the correct order of the first three horses to finish the race. How many different such predictions can be made?

(iii) A person selects two of the seven horses at random. What is the probability that the selected horses are the first two horses to finish the race?

3. (a) Determine the coefficient of x^3y^3 in $(2x-y)^6$.

(b) Prove that if n is a positive integer, then

$$\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n.$$

(c) Five points are marked on a plane. No three of them are collinear. How many different triangles can be formed using these points as vertices?

Two of the five points are labelled X and Y respectively. How many of the above triangles have XY as a side?

4. (a) Differentiate with respect to x:

(i) $\ln\left(\dfrac{3+x}{\sqrt{9-x^2}}\right)$; (ii) $e^x\cos x$; (iii) $\sin(3x^2)$; (iv) $6x^2+3+\dfrac{1}{x^2}$.

(b) Find the slope of the tangent to the curve $x^2+y^3=x-2$ at the point $(3,-2)$.

p.t.o.

5. (a) Find the local maxima and minima of the function $f(x) = e^{-x^2}$, and sketch its graph. [Do **NOT** use graph paper.]

(b) A car moving in a straight line begins to slow down at a point P in order to stop at a red traffic light at Q. The distance of the car from P, after t seconds, is given by

$$s = 12t - \frac{3}{2}t^2$$

where s is in metres.

(i) Find the speed of the car as it passes P.

(ii) Find the time taken to stop.

(iii) The car stops exactly at Q. Find the distance from P to Q.

6. (a) Evaluate the following integrals:

(i) $\displaystyle\int xe^{4x}\,dx$; (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x}\,dx$;

(iii) $\displaystyle\int_0^2 \frac{x+1}{x^2+2x+2}\,dx$; (iv) $\displaystyle\int_0^2 \frac{x^2+2x+2}{x+1}\,dx$.

(b) Find the area enclosed between the parabola $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2010

MATHEMATICS

First Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Let α, β and γ be the three roots of the equation

$$x^3 - 3x^2 + 4x + 5 = 0.$$

Determine the equation whose roots are $\beta + \gamma, \gamma + \alpha, \alpha + \beta$.

(b) Solve the equation

$$x + \sqrt{10 - x^2} = \sqrt{2}$$

for $(-\sqrt{10} < x < \sqrt{10})$.

2. (a) For $x > \dfrac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5 \left(\frac{3}{2x - 1} \right).$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x + 3}{x - 4} < -2.$$

(c) Let a, b be **positive** real numbers. Set $A = \dfrac{a + b}{2}$, $G = \sqrt{ab}$ and $H = \dfrac{2ab}{a + b}$.
(i) Show that $G = \sqrt{AH}$.
(ii) **Prove** that $A - G \geq 0$ with equality if and only if $a = b$.
(iii) Deduce that $A > G > H$ unless $a = b$ in which case show that $A = G = H$.

p.t.o.

3. (a) Let L be the line $y+5=0$ and M be the line $x-1=0$. Verify that both lines are at a (perpendicular) distance 2 from the point $(3,-3)$. Find the equation of the line Q through $(1,-2)$ which is different from M but also a (perpendicular) distance 2 from $(3,-3)$. Find the equation of the line which halves the angle formed by the lines L and Q.

(b) Let C be the circle with centre at $(3,-3)$ and L, M, Q as tangents. Verify that the area of C is greater than twice the area of the triangle formed by L, M, Q.

4. (a) Solve the set of simultaneous equations

$$\begin{array}{rcrcrcl} 3x & + & y & - & z & = & 2 \\ 2x & + & 3y & + & z & = & 7 \\ 4x & - & y & + & 3z & = & 8 \end{array}$$

(b) There are three distinct points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these seven points as vertices?

5. (a) Show that

$$\frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha + i\sin\alpha - 1} = -i\cot\frac{\alpha}{2}$$

(where $i=\sqrt{-1}$).

(b) Show that the complex numbers z $(=x+iy)$ which satisfy

$$|z-1-2i| = 2|z+1-2i|$$

lie on a circle. Find the centre and radius of this circle.

(c) Find also the set of complex numbers z which satisfy

$$|z-1-2i| = |z+1-2i|.$$

6. Find the rational numbers a, b such that

$$f(x) = \frac{1}{(2x+3)(2x+5)} = \frac{a}{2x+3} + \frac{b}{2x+5}.$$

Let x take the values $0, 1, 2, ..., n$.
Compute $f(0)+f(1)+...+f(n)$. Deduce that the area between the curve $y=f(x)$ and the positive x-axis is at most $\frac{1}{6}$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2010

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Find the equation of the line parallel to $3x - 2y = 4$ that also passes through the point of intersection of the lines $2x - y + 6 = 0$ and $5x + 3y - 4 = 0$.

(b) (i) Prove that the measure of one of the angles between two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

(ii) What is the measure of the other angle (in terms of θ)?

(iii) Find the equations of the two lines that pass through the point $(6, 1)$ and make an angle of 45° with the line $x + 2y = 0$.

(c) A circle has the line $y = 2x$ as a tangent at the point $(2, 4)$. The circle also passes through the point $(4, -2)$. Find the equation of the circle.

2. (a) How many different 14–letter "words" can be made with the letters of the word

INDIVISIBILITY ?

Of these "words", how many of them have all the vowels together?

(b) A committee of 8 people is to be selected from 10 men and 10 women.
How many different committees can be formed if:

(i) there are **no** restrictions?
(ii) there must be an equal number of men and women?
(iii) there must be an even number of women?
(iv) there must be more women than men?
(v) there must be at least 1 man?

3. (a) Prove that if n is a positive integer, then

$$\binom{2n}{n} + \binom{2n}{n-1} = \frac{1}{2}\binom{2n+2}{n+1}$$

(b) Determine the coefficient of x^4y^3 in $(2x^2 - 3y)^5$.

(c) In a café there are 11 seats in a row at the counter. Six people are seated at random at the counter.
How much more likely is it that all six are seated together than that no two of them are seated together?

p.t.o.

4. (a) Differentiate with respect to x:

$$(i)\ \frac{x^3}{\sqrt{6-x^2}} \quad (ii)\ \cos^6(4x^3+3) \quad (iii)\ e^{2x^5} \quad (iv)\ 7t^2-4t+5$$

(b) A curve is defined by the equation $x^2y^3+4x+2y=12$.

(i) Find $\frac{dy}{dx}$ in terms of x and y.

(ii) Show that the tangent to the curve at the point $(0,6)$ is also the tangent to it at the point $(3,0)$.

5. (a) Find the local maxima and minima of the function $f(x)=\frac{2x^2-4x+4}{x-2}$, and sketch its graph. [Do **NOT** use graph paper]

(b) An object moves in a straight line such that its distance from a fixed point is given by $s=\sqrt[3]{t^2+2}$, where s is in metres and t is in seconds.

(i) Find the speed of the object when $t=5$ seconds.

(ii) Find the acceleration of the object when $t=5$ seconds.

6. (a) Evaluate the following integrals:

$$(i) \int \left(\sin 2x + e^{4x}\right)\, dx \quad (ii) \int_0^{\pi} x\cos x\, dx \quad (iii) \int \sin^4 x \cos^3 x\, dx$$

(b) Find the area enclosed between the parabola x^2-6x+7 and the line $y=x-3$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2009

MATHEMATICS

First Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) One root of the equation

$$2x^3 - x^2 + 2x - 1 = 0$$

is i, where $i = \sqrt{-1}$. Find the other two roots of the equation.

(b) Find all values of the real number a for which the quadratic equation

$$x^2 + ax + 4 = 0$$

has real roots.
For what values of a is there a complex root with imaginary part equal to i?

2. (a) The numbers

$$3 + x, \quad 5x - 1, \quad 1 + 3x$$

are the first three terms in a **geometric** progression. Find the values of x and the sum of the first n terms of each of the corresponding series.

(b) Let $h(x) := 3x^2 - x$. Show that the sum of the twenty numbers $h(1), h(2), h(2^2), \ldots, h(2^{19})$ is equal to

$$2^{20}(2^{20} - 1).$$

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5,5)$, $(4,8)$, $(-7,-3)$, the co–ordinates of its centre and the length of the radius.

(b) Find the equation of the circle which touches the y–axis at the point $(0, 3)$ and passes through the point $(1, 0)$. Hence find the co–ordinates of its centre and the length of its radius.

(c) Find the radius and centre of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $3x+4y=4$.

4. (a) For $x > \frac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5 \left(\frac{3}{2x-1}\right).$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x+3}{x-4} < -2.$$

(c) Let a, b be **positive** real numbers. Set $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.
(i) Show that $G = \sqrt{AH}$.
(ii) **Prove** that $A - G \geq 0$ with equality if and only if $a = b$.
(iii) Deduce that $A > G > H$ unless $a = b$ in which case show that $A = G = H$.

5. (a) Prove that

$$\frac{\sin\theta}{1+\cos\theta} = \tan(\theta/2).$$

For $x \geq 0$, if $\sin\theta = \frac{1-x}{1+x}$, express $\cos\theta$ in terms of x. Hence, or otherwise show that

$$\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{x}.$$

(b) Simplify the equation

$$\cos\left(\phi - \frac{\pi}{6}\right) - 3\sin\left(\phi + \frac{\pi}{3}\right) = 1,$$

and hence find one value of ϕ which satisfies it.

p.t.o.

6. (a) Determine k such that

$$(1 + kn + 2n^2)(1 - kn + 2n^2) = 1 + 4n^4.$$

Hence or otherwise find real numbers a and b such that

$$\frac{4n}{1+4n^4} = \frac{a}{1+kn+2n^2} + \frac{b}{1-kn+2n^2}.$$

(b) Using part (a) or otherwise, calculate

$$s_n := \sum_{k=1}^{n} \frac{4k}{1+4k^4}\ .$$

By finding $\lim_{n\to\infty} s_n$, deduce that

$$\sum_{k=1}^{\infty} \frac{4k}{1+4k^4} = 1\,.$$

(c) Find the **least** value of r such that

$$\sum_{k=1}^{r} \frac{4k}{1+4k^4} > \frac{999}{1000}$$

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2009

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Find the equation of the line through the point $(1, 0)$ that also passes through the point of intersection of the lines $2x - y + 6 = 0$ and $10x + 3y - 2 = 0$.

(b) (i) Prove that the measure of one of the angles between two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

(ii) Find the equations of the two lines that pass through the point $(6, 1)$ and make an angle of 45° with the line $x + 2y = 0$.

(c) S is the circle $x^2 + y^2 + 4x + 4y - 17 = 0$ and K is the line $4x + 3y = 12$.

(i) Show that the line K does not intersect S.

(ii) Find the co-ordinates of the point on S that is closest to K.

2. (a) How many different 13–letter "words" can be made with the letters of the word

INTELLECTUALS ?

Of these "words", how many of them have all the vowels together?

(b) A committee of 10 people is to be selected from 10 men and 10 women.
How many different committees can be formed if:

(i) there are **no** restrictions?
(ii) there must be an equal number of men and women?
(iii) there must be an even number of women?
(iv) there must be more women than men?
(v) there must be at least 7 men?

3. (a) (i) In how many different ways can eight people be seated in a row?

(ii) Three girls and five boys sit in a row, arranged at random. Find the probability that the three girls are seated together.

(iii) Three girls and n boys sit in a row, arranged at random. If the probability that the three girls are seated together is $\frac{1}{35}$, find the value of n.

(b) Prove that if n is a positive integer, then

$$\binom{2n}{n} + \binom{2n}{n-1} = \frac{1}{2}\binom{2n+2}{n+1}$$

(c) Determine the coefficient of x^9y^3 in $(2x - 3y)^{12}$.

p.t.o.

4. (a) Differentiate with respect to x:

$$(i)\,\frac{x^3}{\sqrt{6-x^2}} \quad (ii)\,\tan^6(4x^3+3) \quad (iii)\,e^{7x^3} \quad (iv)\,\cos^2 y+\sin^2 y$$

(b) Determine which points on the curve $y = 4 - x^2$ are closest to the point (0, 2).

5. (a) Find the local maxima and minima of the function $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$, and sketch its graph. [Do **NOT** use graph paper]

(b) An object moves in a straight line such that its distance from a fixed point is given by $s = \sqrt{t^2 + 1}$, where s is in metres and t is in seconds.

(i) Find the speed of the object when $t = 5$ seconds.

(ii) Find the acceleration of the object then $t = 5$ seconds.

6. (a) Evaluate the following integrals:

$$(i)\int\left(6x+3+\frac{1}{x^2}\right)dx \quad (ii)\int_0^{\pi} x\sin x\,dx \quad (iii)\int \sin^3 x\cos^4 x\,dx$$

(b) Find the area between the curve (parabola) $y = x^2$ and the line $y = 2 - x$.

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COLLEGE OF ENGINEERING AND INFORMATICS

ENTRANCE EXAMINATION 2008

MATHEMATICS

First Paper

Time allowed: *Two* hours.
Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Solve
$$4x^{2/5} - 10x^{1/5} - 6 = 0$$
(b) Given that $x = 3$ is a root of the equation
$$3x^3 - 18x^2 + 33x - 18 = 0$$
determine the other two roots.
(c) Solve
$$\sqrt{2x+3} + \sqrt{x-2} = 4$$

2. (a) Show that
$$\cos(3\theta) = 4\cos\theta - 4\cos\theta\sin^2\theta - 3\cos^2(\theta/2) + 3\sin^2(\theta/2).$$
(b) Show that
$$\sin^4\theta = \frac{3 - 4\cos 2\theta + \cos 4\theta}{8}.$$

3. (a) If $ax^2 + by^2 + cxy + dx + ey + f = 0$ is the equation of a circle with radius 5 and center at (1,3), determine the values of a, b, c, d, e, f.
(b) Calculate the equation of the straight line between the point (1,1) and the center of the circle $2x^2 + 2y^2 + 4x + 3y = 3$.
(c) Suppose we have a square, whose side length is 2 and upper-right corner (vertex) is at (3,4). Calculate the equation of the circle that passes through the 4 corners (vertices) of the square.

P.T.O.

4. (a) The effective broadcast area of a radio station is bounded by the circle $x^2 + y^2 = 2,500$ where x and y are measured in miles. Another radio station's broadcast area is bounded by the circle $(x - 100)^2 + (y - 100)^2 = 900$. Determine (with an explanation) if there is any location that can receive both stations.

   (b) Solve the set of simultaneous linear equations

$$\begin{aligned} x + 2y + z &= 8 \\ 2x + y - z &= 1 \\ x + y - 2z &= -3 \end{aligned}$$

5. (a) Calculate all the roots of $z^4 + 16 = 0$, where z is a complex number.

   (b) Evaluate $(1 - i\sqrt{3})^{-6}$ where $i = \sqrt{-1}$.

   (c) Simplify

$$\frac{(3 - 3i)^4}{(\sqrt{3} + i)^3}.$$

6. (a) Determine whether the following sequences (given by their general term T_n) converge or diverge, and in the case of convergence, calculate $\lim_{n\to\infty} T_n$.

   (i)

$$T_n = (-1)^{n+1}(\frac{n-1}{n})$$

   (ii)

$$T_n = \frac{\cos n}{n}$$

   (iii)

$$T_n = (-1)^{\frac{4n+1}{2}}(\frac{n-1}{n})$$

   (b) How many terms of the series $1 + 3 + 9 + 27 + \ldots$ are required for the sum to be greater than 20 terms of the series $10 + 20 + 40 + 80 + \ldots$?

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

COLLEGE OF ENGINEERING AND INFORMATICS

ENTRANCE EXAMINATION 2008

MATHEMATICS

Second Paper

Time allowed: *Two* hours.

Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Write down the third and fifth terms in the expansion of

   (i) $(3x - 2y)^6$

   (ii) $(a + b)^9$

   (b) Solve the following equation for n:

$$\frac{n!}{(n-2)!} = 9,900$$

   (c) Prove that, for $n > 0$,

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

2. (a) Write down the repeating decimal $1.24123123123\ldots$ as a rational number.

   (b) Determine the following sums:

   (i)

$$\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$$

   (ii)

$$\sum_{n=1}^{\infty} \frac{40n}{(2n-1)^2(2n+1)^2}$$

3. (a) In how many different ways can 5 people be seated at a round table?

   (b) A class consists of 15 men and 8 women. In how many different ways can a tag-rugby team be chosen with 3 men and 3 women?

   (c) In how many different ways can the letters in the word *number* be arranged if the e and r cannot be side by side?

P.T.O.

4. (a) Differentiate with respect to x

$$\text{(i) } \tan(5 - \sin 2x) \qquad \text{(ii) } \frac{1}{21}(3x-2)^7 + (4 - \frac{1}{2x^2})^{-1} \qquad \text{(iii) } e^{\sin 2x - \frac{3}{x^4}}$$

(b) Water runs in to a conical tank at the rate of 2 cubic metres per minute. The tank stands point down and has a height of 3 metres and a base radius of 1.5 metres. How fast is the water level rising when the water is 2 metres deep?

5. A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, while the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed (call it P). Find the proportions of the window (specifically: the ratio of the height to the width of the rectangular part) that will admit the most light.

6. (a) Evaluate the following integrals:

$$\text{(i) } \int \frac{e^{-\sqrt{r}}}{\sqrt{r}} dr \quad \text{(ii) } \int_0^1 \frac{36dx}{(2x+1)^3} \quad \text{(iii) } \int \frac{\theta^2}{9\sqrt{(73+\theta^3)}} d\theta \quad \text{(iv) } \int_{-1}^1 2x \sin(1 - x^2) dx$$

(b) Find the area between the curve $y = x^4$ and the line $y = 8x$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2007

MATHEMATICS

First Paper

Time allowed: *Two* hours.
Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Is there a number x which is one less than its cube? If so, find the value of x accurate to one decimal place.

   (b) Given that $x = 1$ is a root of the equation

   $$2x^3 - 5x^2 - 6x + 9 = 0$$

   determine the other two roots.

   (c) By setting $y = x^2$ or otherwise, find all the roots of the equation $x^4 - 29x^2 + 100 = 0$.

2. (a) Show that

   $$\frac{1}{\sin\theta\cos\theta} = \tan\theta + \cot\theta.$$

   (b) Show that

   $$\cos(\frac{\pi}{3} + \theta) + \cos(\theta - \frac{\pi}{3}) = \frac{\sin\theta}{\tan\theta}.$$

3. (a) C is the circle $x^2+y^2-6x-4y-36 = 0$ of centre p. K is the circle $x^2+y^2-6x-8y-11 = 0$ of centre q. Calculate the equation of the straight line between p and q.

   (b) Calculate the mid-point between (7,9) and the center of the circle $x^2+y^2-4x-4y+4 = 0$.

   (c) Determine if the points $(5, 4), (-1, 3)$ and $(-2, -7)$ are inside, on or outside the circle $x^2 + y^2 - 6x - 4y - 36 = 0$.

P.T.O.

4. (a) Find whether, and if so where, the lines (a) $y = 2x - 4$, (b) $3y = x + 11$, (c) $y = 3x + 6$ cut the circle $x^2 - 4x + y^2 - 2y - 5 = 0$.

   (b) Solve the set of simultaneous linear equations

$$\begin{aligned} x + 3y + z &= 8 \\ 2x + y + 3z &= 7 \\ x + y - z &= 2 \end{aligned}$$

5. (a) Calculate all the roots of $z - i^{-1/3} = 0$, where z is a complex number and $i = \sqrt{-1}$.

   (b) Evaluate z^{20} when $z = \sqrt{3} + i$.

   (c) Find $(\frac{1}{2} + \frac{1}{2}i)^{10}$.

6. (a) For each of the following sequences, determine $\lim_{n\to\infty} T_n$, where T_n is

   (i)
$$\frac{1 + 2^2 + 3^2 + 4^2 + \ldots + n^2}{n^2}$$

   (ii)
$$\frac{5^{n+1} + 7^{n+1}}{5^n - 7^n}$$

   (iii)
$$\frac{n^2}{\sqrt{2n^4 + 1}}$$

   (b) How many terms of the series $1 + 2 + 4 + 8 + \ldots$ are required for the sum to be greater than a million?

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2007

MATHEMATICS

Second Paper

Time allowed: *Two* hours.
Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Write down and simplify the first four terms in the expansion of
   (i) $(\frac{1}{2}x + 3y)^{16}$
   (ii) $(2x - y)^{12}$

   (b) Prove that

$$(r+1)\binom{n}{r+1} + r\binom{n}{r} = n\binom{n}{r}$$

   where

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}.$$

   (c) Prove that

$$\binom{n}{0}3^n + \binom{n}{1}3^{n-1} + \binom{n}{2}3^{n-2} + \ldots + \binom{n}{n}3^0 = 4^n.$$

2. (a) Write down the repeating decimal $2.31717171717\ldots$ as a rational number.

   (b) Determine the following sums:

   (i)

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$$

   (ii)

$$\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{4^n}$$

P.T.O.

3. How many different 8 letter "words" can be formed from the letters of the word GAILLIMH? Explain your answer. How many of these "words"

   (a) have the letter G after the letter H (not necessarily immediately after)?

   (b) have A and M beside one another (in either order - AM or MA)?

   (c) have all the vowels next to one another?

   (d) begin and end with the letter L?

4. (a) Differentiate with respect to x

$$\text{(i) } \frac{2-x^2}{x^3+x} \qquad \text{(ii) } \frac{\sin(2-x)}{\cos^2(\sqrt{x})} \qquad \text{(iii) } e^{\sin^2 x - \cos x}$$

   (b) Determine which points on the graph of $x^2 + y - 10 = 0$ are closest to the point $(0, 0)$.

5. (a) A frame is to be made for a painting which will be in the shape of a rectangle surmounted by a semicircle (the length of the diameter of the semicircle equals the width of the rectangle). Five metres of wood is available to make the frame. How should the frame be constructed to enclose the greatest possible area? Calculate this (maximum) area (in square metres).

6. (a) Evaluate the following integrals:

$$\text{(i) } \int x^2 \sin x \, dx \quad \text{(ii) } \int_{1/2}^{2/3} \frac{dx}{x^2+4x-12} \quad \text{(iii) } \int \cos^3 x \, dx \quad \text{(iv) } \int_0^{\pi/4} \cos\theta \sqrt{\sin\theta + 1} \, d\theta$$

   (b) Find the area between the curve (parabola) $y = 3x^2/4$ and the line $y = 4x - 1$.

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NATIONAL UNIVERSITY OF IRELAND, GALWAY

FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2006

MATHEMATICS

First Paper

Time allowed: *Two* hours.
Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Is there a number x which is one less than its cube? If so, find the value of x accurate to one decimal place.

   (b) Given that $x = -3$ is a root of the equation

   $$x^3 + 3x^2 - 7x - 21 = 0$$

   determine the other two roots.

   (c) Find all values of k for which $x = 1$ is a root of the equation

   $$k^2x^3 - 7kx + 10 = 0.$$

2. (a) Show that

   $$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta.$$

   (b) Show that

   $$\cos(\frac{\pi}{3} + \theta) + \cos(\frac{\pi}{3} - \theta) = \cos\theta.$$

3. (a) C is the circle $x^2+y^2-6x-4y-36 = 0$ of centre p. K is the circle $x^2+y^2+10x+4y+20 = 0$ of centre q. Calculate the mid-point between p and q.

   (b) Calculate the distance from the point (7,8) to the center of the circle $x^2+y^2-4x-4y+4 = 0$.

   (c) Determine if the points $(5, 3), (-1, 4)$ and $(-2, -3)$ are inside, on or outside the circle $x^2 + y^2 - 6x - 8y - 11 = 0$.

P.T.O.

4. (a) Consider two points p_1 and p_2 on a circle C_1 and two points q_1 and q_2 on a circle C_2. C_1 does not intersect C_2. Additionally, the line through q_1, q_2 does not intersect C_1, but the line through p_1, p_2 intersects C_2 at q_1 and also passes through the center c_2 of C_2. Determine how many different triangles can be made from the five points p_1, p_2, q_1, q_2, c_2.

(b) Solve the set of simultaneous linear equations

$$\begin{aligned} y - 3z &= -5 \\ 2x + 3y - z &= 7 \\ 4x + 5y - 2z &= 10 \end{aligned}$$

5. (a) Find the four fourth roots of -16 (i.e. the four possible values of z such that $z^4 = -16$).

(b) Write

$$\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}}$$

in the form $re^{i\theta}$ where $r \geq 0$ and $-\pi \leq \theta \leq \pi$.

(c) Write

$$\frac{z^{10} - z^9}{z + i}$$

in the form $x + iy$, where x and y are real numbers, and $z = 1 + i$.

6. (a) For each of the following sequences, determine $\lim_{n\to\infty} T_n$, where T_n is

(i) $n/(2n + 1)$

(ii) $10^n/n!$

(iii) $3 - 1/n$

(b) Determine

$$\sum_{k=0}^{\infty} \frac{5}{4^k}.$$

(c) Find the rational number represented by the repeating decimal 0.784784784...

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ENTRANCE EXAMINATION 2006

MATHEMATICS

Second Paper

Time allowed: *Two* hours.

Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Show that no term in the expansion of

$$\left(\frac{2}{\sqrt{x}} - \frac{7}{\sqrt[3]{x}}\right)^{20}$$

is independent of x. Determine the coefficient of x^{-8} in the expansion.

(b) Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^n \binom{n}{n} = 0$$

where

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} \text{ and } 0! = 1.$$

2. Determine the following sums:

(a)

$$\sum_{k=1}^{\infty} 3^{2k} 5^{1-k}$$

(b)

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

(c)

$$\sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}}$$

P.T.O.

3. How many different 10 letter "words" can be formed from the letters of the word BRATISLAVA? Explain your answer. How many of these "words"

   (a) have two A's together?

   (b) have three A's together?

   (c) begin and end with the letter A?

   (d) begin with the three A's?

4. (a) Differentiate with respect to x

$$\text{(i) } \sqrt[3]{\frac{x-1}{3+x^2}} \qquad \text{(ii) } \sin^3(\sqrt{x})\cos(x^3) \qquad \text{(iii) } \frac{1}{e^{\sin(x)}}$$

   (b) Determine which points on the graph of $y = 8 - x^2$ are closest to the point $(1, -3)$.

5. (a) A closed cylindrical can is to hold one litre (1,000 cubic centimetres) of liquid. How should we chose the height and radius of the can to minimize the amount of material needed to make it? If instead we consider a cylindrical can which is open at the top, what is the result?

6. (a) Evaluate the following integrals:

$$\text{(i) } \int x\cos x\,dx \quad \text{(ii) } \int_0^{\pi/4} \cos\theta\sqrt{\sin\theta+1}\,d\theta \quad \text{(iii) } \int \sin^3 x\,dx \quad \text{(iii) } \int_{3/4}^{4/5} \frac{dx}{x^2-5x+6}$$

   (b) Find the area between the curve (parabola) $y = 2x^2/3$ and the line $y = 4x - 1$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2005

MATHEMATICS

First Paper

Time allowed: *Two* hours.

Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Find the roots α, β, γ of the equation

$$x^3 - 15x^2 + 66x - 80 = 0$$

given that the roots are in arithmetic progression.

(b) Let α and β be the two roots of the equation

$$x^2 - 6x + 3 = 0.$$

Determine the value of $\alpha^3 + \beta^3$.

2. (a) Show that

$$\frac{\cos\theta - 1}{\sin\theta} = -\tan(\theta/2)$$

(b) Show that

$$\cos 4\theta - 1 = 8\sin^4\theta - 8\sin^2\theta$$

3. (a) C is the circle $x^2+y^2-6x-8y-11 = 0$ of centre p. K is the circle $x^2+y^2+10x+4y+20 = 0$ of centre q. Calculate $|pq|$.

(b) A line containing the point P with coordinates $(7, 6)$ touches the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ at the point Q. Calculate the distance PQ.

(c) Determine if the points $(5, 3), (-1, 4)$ and $(-2, -3)$ are inside, on or outside the circle $x^2 + y^2 - 6x - 4y - 7 = 0$.

P.T.O.

4. (a) There are four points on a line L and five points on a circle C. L and C do not intersect, and none of the four points on L are colinear with any pair of the five points on C. What is the greatest number of triangles that can be formed from any three of these nine points?

   (b) Solve the set of simultaneous linear equations

$$\begin{aligned} 4x + y - 2z &= 3 \\ -x - y + z &= 3 \\ -4x - 2y + 3z &= -2 \end{aligned}$$

5. (a) By writing $z = \sqrt{3} + i$ (where $i = \sqrt{-1}$) in modulus-argument form (or otherwise), calculate $(z/2)^{2005}$.

   (b) Find $\sqrt{-5+12i}$ in the form $x + iy$, where x and y are real numbers.

   (c) Write

$$\frac{z^{11} - z^{10}}{z - i}$$

   in the form $x + iy$, where x and y are real numbers, and $z = 1 - i$.

6. (a) Find rational numbers a and b such that

$$\frac{1}{(k+5)(k+7)} = \frac{a}{k+5} + \frac{b}{k+7}$$

   Hence (or otherwise) calculate

$$S_n = \sum_{k=0}^{n} \frac{1}{(k+5)(k+7)}$$

   By finding $\lim_{n\to\infty} S_n$, deduce that

$$\sum_{k=0}^{\infty} \frac{1}{(k+5)(k+7)} = \frac{11}{60}$$

   (b) Find the least value of r such that

$$\sum_{k=0}^{r} \frac{1}{(k+5)(k+7)} > \frac{1}{6}$$

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NATIONAL UNIVERSITY OF IRELAND, GALWAY

FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2005

MATHEMATICS

Second Paper

Time allowed: *Two* hours.

Candidates for IT courses should do *four* questions. All other candidates should do *five* questions.

DO NOT USE GRAPH PAPER

1. (a) Calculate whether or not any term in the expansion of

$$\left(\sqrt{x} - \frac{3}{x^{5/2}}\right)^{10}$$

is independent of x. Determine the coefficient of x^{-7} in the expansion.

(b) Prove that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n$$

where

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} \text{ and } 0! = 1.$$

2. (a) A ball is dropped from a height of 5 metres and begins bouncing. Each bounce is 2/3 the height of the previous bounce. Find the total vertical distance travelled by the ball.

(b) Consider the sequence a_n where

$$a_n = \frac{3 - (k^2 + 1)n}{2 - kn}$$

and k is a constant. Find the limit of the sequence when k is two. Also, find k when the limit of the sequence is 2.

P.T.O.

3. (a) How many different 11 letter "words" can be formed from the letters of the word MATHEMATICS? Explain your answer. How many of these "words" have the two T's together? How many have two A's followed by two M's?

(b) If repetition is not allowed

(i) How many 3-digit numbers can be formed from the set $\{2, 3, 4, 5, 6, 7, 8\}$?

(ii) How many of these are even numbers?

(iii) How many are odd numbers between 500 and 600?

4. (a) Differentiate with respect to x

$$\text{(i)}\ \sqrt{\frac{x-1}{3+x^2}} \qquad \text{(ii)}\ \sin^3(4x)\cos(x) \qquad \text{(iii)}\ \frac{1}{\cos(\ln x)}$$

(b) Determine which points on the graph of $y = 4 - x^2$ are closest to the point (0,0) (i.e. the origin).

5. (a) A frame is to be made for a painting which will be in the shape of a rectangle surmounted by an equilateral triangle (the length of the base of the triangle equals the width of the rectangle). Five metres of wood is available to make the frame. How should the frame be constructed to enclose the greatest possible area? Calculate this (maximum) area (in square metres).

6. (a) Evaluate the following integrals:

$$\text{(i)}\ \int x\sin x\,dx \qquad \text{(ii)}\ \int_0^2 x^4\sqrt{x^5+1}\,dx \qquad \text{(iii)}\ \int_0^{\pi/4} \sin^3 x\,dx \qquad \text{(iii)}\ \int \frac{dx}{x^2-7x+10}$$

(b) Find the area between the curve (parabola) $y = 2x^2$ and the line $y = 5x - 3$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2004

MATHEMATICS

First Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Let α, β and γ be the three roots of the equation

$$x^3 - 3x^2 + x + 2 = 0.$$

Determine the value of $\alpha^3 + \beta^3 + \gamma^3$.

(b) Find the roots α, β, γ of the equation

$$3x^3 - 26x^2 + 52x - 24 = 0.$$

given that the roots are in geometric progression.

2. (a) Prove that

$$\tan\frac{\pi}{12} = 2 - \sqrt{3}$$

(b) Let α, β, γ be the three angles of an arbitrary triangle. Prove that

$$\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma.$$

(c) For a given constant k, find all values of θ which satisfy

$$\cos k\theta - \cos(k+2)\theta = \sin\theta$$

p.t.o.

3. (a) Find the equation of the circle passing through the three points $(5, 5)$, $(4, 8)$, $(-7, -3)$, the co-ordinates of its centre and the length of the radius.

(b) Find the equation of the circle which touches the y-axis at the point (0, 3) and passes through the point (1, 0). Hence find the co-ordinates of its centre and the length of its radius.

(c) Find the radius and centre of the two circles in the first quadrant touching both the x-axis and the y-axis and the line $3x + 4y = 4$.

4. (a) For $x > \frac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5 \left(\frac{3}{2x-1}\right).$$

(b) For $x \neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x+3}{x-4} < -2.$$

(c) Let a, b be **positive** real numbers.

Set $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.
(i) Show that $G = \sqrt{AH}$.
(ii) **Prove** that $A - G \geq 0$ with equality if and only if $a = b$.
(iii) Deduce that $A > G > H$ unless $a = b$, in which case show that $A = G = H$.

p.t.o.

5. (a) Write the complex number $w = -8 + 8\sqrt{3}i$ in modulus–argument form, ie in the form $r(\cos\theta + i\sin\theta)$ (where $i = \sqrt{-1}$).

(b) Using De Moivre's Theorem or otherwise, show that $z = \sqrt{3} + i$ satisfies $z^4 = -8 + 8\sqrt{3}i$.
Find the other three roots of the equation

$$z^4 = -8 + 8\sqrt{3}i.$$

(c) Find the square roots of the complex number $-5 + 12i$

6. (a) Find real numbers a and b such that

$$\frac{1}{4k^2-1} = \frac{a}{2k-1} + \frac{b}{2k+1}.$$

Hence, or otherwise calculate

$$s_n := \sum_{k=1}^{n} \frac{1}{4k^2-1} \,.$$

By finding $\lim_{n\to\infty} s_n$, deduce that

$$\sum_{k=1}^{\infty} \frac{1}{4k^2-1} = \frac{1}{2}$$

(b) Find the least value of r such that

$$\sum_{k=1}^{r} \frac{1}{4k^2-1} > \frac{99}{200}$$

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2004

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Find the term independent of x, ie the constant term, in the binomial expansion of

$$\left(\sqrt{x} - \frac{2}{x^2}\right)^{10}.$$

(b) From the expansion of $\left(1 + \frac{2x}{3}\right)^{12}$ find

(i) the value of the greatest coefficient

(ii) the value of the greatest term when $x = \frac{3}{4}$.

2. (a) How many different 11–letter "words" can be made with the letters of the word

NINCOMPOOPS ?

Of these "words", how many of them have all the vowels together?

p.t.o.

(b) A committee of 8 people is to be selected from 10 men and 8 women.
How many different committees can be formed if:
(i) there are no restrictions?
(ii) there must be an equal number of men and women?
(iii) there must be an even number of women?
(iv) there must be more women than men?
(v) there must be at least 7 men?

3. (a) Prove that if n is a positive integer, then

$$\binom{2n}{n} + \binom{2n}{n-1} = \frac{1}{2}\binom{2n+2}{n+1}$$

(b) Prove that for any positive integer n

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots \pm \binom{n}{n} = 0$$

(c) (i) In how many ways can 5 different postcards be sent to three people?
(ii) In how many ways can 3 different postcards be sent to five people?
(iii) In how many ways can 3 different postcards be sent to five people such that no person gets more than one postcard?

4. (a) Differentiate with respect to x:

$$(i)\ \frac{x^4}{\sqrt{6-x}} \quad (ii)\ \cos^3(x^4+3) \quad (iii)\ (\sin^{-1} x)^3 \quad (iv)\ 2x\ln x$$

(b) Determine which points on the curve $y = 4 - x^2$ are closest to the point (0, 2).

V

5. (a) Find the local maxima and minima of the function $f(x) = 2x^3 - 3x^2 - 12x + 11$, and sketch its graph. [Do **NOT** use graph paper]
How many real roots does $f(x) = 0$ have?
(b) Find the equation of the tangent to $f(x)$ at the point $x = 0$.

6. (a) Evaluate the following integrals:

$$(i) \int_0^\pi x \cos x \, dx \quad (ii) \int \sin^4 x \cos^3 x \, dx \quad (iii) \int \frac{dx}{x^2 - 7x + 12}$$

(b) Find the area between the curve (parabola) $y = x^2$ and the line $y = 2 - x$.

V

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2003

MATHEMATICS

First Paper

Time allowed: *Two* hours

Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Let α and β be the two roots of the equation

$$x^2 - 6x + 2 = 0.$$

Determine the value of $\alpha^3 + \beta^3$.

(b) Let α and β and γ be the roots of

$$x^3 + x^2 + x + 1 = 0.$$

Prove that

$$\alpha^{2003} + \beta^{2003} + \gamma^{2003} = \alpha + \beta + \gamma.$$

2. (i) Show that

$$\frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$$

(ii) If $\sin\theta = \dfrac{1-x}{1+x}$ $(x \geq 0)$, express $\cos\theta$ in terms of x and hence show that

$$\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \sqrt{x}.$$

(iii) Simplify the equation

$$\cos\left(y - \frac{\pi}{6}\right) - 3\sin\left(y + \frac{\pi}{3}\right) = 1$$

and hence (or otherwise) find one value of y which satisfies it.

p.t.o.

3. (a) The equation of a circle is

$$(x+9)(x+3)+(y-2)(y+2)=0$$

Find the centre of the circle and the length of its radius.

(b) The x-axis is a tangent to the circle

$$x^2+y^2+2gx+2fy+c=0.$$

Prove that $g^2=c$.

(c) The x-axis is tangent to a circle at the point $(3, 0)$. The point $(-1, 4)$ is on the circle. Find the centre of the circle, the equation of the circle and the length of its radius.

4. (a) Solve the simultaneous equations:

$$\begin{aligned} 2x-3y &= 1 \\ x^2+xy-4y^2 &= 2 \end{aligned}$$

(b) For $x>\frac{1}{2}$ solve the equation:

$$\log_5 x = 1+\log_5\left(\frac{3}{2x-1}\right).$$

(c) For $x\neq 4$, determine the values of x for which the following inequality holds:

$$\frac{x+3}{x-4}<-2.$$

5. (a) Show that

$$\frac{\cos\alpha+i\sin\alpha+1}{\cos\alpha+i\sin\alpha-1}=-i\cot\frac{\alpha}{2}$$

(where $i=\sqrt{-1}$).

(b) Show that the complex numbers z $(=x+iy)$ which satisfy

$$|z-1-2i|=2|z+1-2i|$$

lie on a circle. Find the centre and radius of this circle.

(c) Find also the set of complex numbers z which satisfy

$$|z-1-2i|=|z+1-2i|.$$

6. (a) Find real numbers a and b such that

$$\frac{1}{4k^2-1} = \frac{a}{2k-1} + \frac{b}{2k+1}.$$

Hence, or otherwise calculate

$$s_n := \sum_{k=1}^{n} \frac{1}{4k^2-1}.$$

By finding $\lim_{n\to\infty} s_n$, deduce that

$$\sum_{k=1}^{\infty} \frac{1}{4k^2-1} = \frac{1}{2}$$

(b) Find the **least** value of r such that

$$\sum_{k=1}^{r} \frac{1}{4k^2-1} > \frac{99}{200}$$

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2003

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Let the cubic polynomial $f(z)$ have the factorisation:

$$f(z) = (z - i)(z^2 + az + b)$$

where $i = \sqrt{-1}$ and a and b are real numbers. Determine the numbers a, b and hence find the roots of $f(z) = 0$.

(b) For an arbitrary complex number z ($\neq i$) let u equal $\dfrac{z+i}{z-i}$, where $i = \sqrt{-1}$.

i. Find the set of numbers z for which $|u| = 1$.
ii. Show that u is a real number if and only if z is imaginary (excluding $z = i$).
iii. Express z in terms of u.

(c) Write the complex number $1-i$ in modulus–argument form, and using de Moivre's Theorem, or otherwise, find $(1-i)^{16}$.

2. (a) How many different 13–letter "words" can be made with the letters of the word

INTELLECTUALS ?

Of these "words", how many of them have all the vowels together?

p.t.o.

(b) A committee of 10 people is to be selected from 10 men and 10 women. How many different committees can be formed if:

(i) there are no restrictions?

(ii) there must be an equal number of men and women?

(iii) there must be an even number of women?

(iv) there must be more women than men?

(v) there must be at least 7 men?

3. (a) Prove that if n is a positive integer, then

$$\binom{2n}{n} + \binom{2n}{n-1} = \frac{1}{2}\binom{2n+2}{n+1}$$

(b) Prove that for any positive integer n

$$\binom{n}{0} + \binom{n}{2} + \cdots = \binom{n}{1} + \binom{n}{3} + \cdots$$

(c) Determine the coefficient of x^9y^3 in $(2x - 3y)^{12}$.

4. (a) Differentiate with respect to x:

$$(i)\ \frac{x^3}{\sqrt{6-x^2}} \quad (ii)\ \tan^6(4x^3+3) \quad (iii)\ e^{7x^3} \quad (iv)\ \cos^2 y + \sin^2 y$$

(b) Determine which points on the curve $y = 4 - x^2$ are closest to the point (0, 2).

5. (a) Find the local maxima and minima of the function $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$, and sketch its graph. [Do NOT use graph paper]

(b) Show that

$$\lim_{x\to\infty}(f(x) - x) = 0$$

and thus deduce that the line $y = x$ is an asymptote to $f(x)$.

6. (a) Evaluate the following integrals:

$$(i)\ \int_0^{\pi} x \sin x\, dx \quad (ii)\ \int \sin^3 x \cos^4 x\, dx \quad (iii)\ \int \frac{dx}{x^2 - 5x + 6}$$

(b) Find the area between the curve (parabola) $y = x^2$ and the line $y = 2 - x$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2002

MATHEMATICS

First Paper

Time allowed: *Two* hours
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Let α and β be the two roots of the equation:

$$x^2 - 4x + 9 = 0.$$

Determine the value of $\alpha^2 + \beta^2$. Find the equation whose roots are α^3 and β^3.

(b) The integers a, b, c, and n are such that $x = n$ is one root of the equation

$$x^3 + ax^2 + bx + c = 0.$$

Prove that n divides c. Hence, or otherwise, find the three roots of the equation

$$x^3 - 4x^2 + 2x - 161 = 0$$

given that one root is an integer.

2. (a) Find the sum of all the natural numbers less than 2002 which are divisible by 7.

(b) The numbers

$$3 + x, \quad 5x - 1, \quad 1 + 3x$$

are the first three terms in an arithmetic progression. Find the value of x and the sum of the first n terms of the corresponding series.

(c) Show that

$$\frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}.$$

Hence, or othewise, determine

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}.$$

3. (a) Let l be the line

$$y + 5 = 0$$

and k be the line

$$x - 1 = 0.$$

Verify that both lines are a distance 2 from the point $(3, -3)$. Find the equation of the line q through $(1, -2)$ which is different from k, but also a distance 2 from the point $(3, -3)$. Find the equation of the (internal) bisector of the angle between l and q.

(b) Let C be the circle with centre at $(3, -3)$ and l, k, q as tangents. Verify that the area of C is greater than twice the area of the triangle formed by the three lines l, k and q.

4. (a) Express $a \cos\phi + b \sin\phi$ in the form $k \cos(\phi - \alpha)$ where k depends on a and b.

(b) For what values of x does the equation $a \cos\phi + b \sin\phi = x$ have a solution? Hence or otherwise solve the equation

$$\cos\phi + \sqrt{3} \sin\phi = 1 \text{ for } 0 < \phi < 2\pi.$$

(c) If $\alpha + \beta + \gamma = 180°$, prove that $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$.

5. (a) Writing $1 - i$, where $i = \sqrt{-1}$ in modulus–argument form (or otherwise), calculate $(1 - i)^{2002}$.

(b) Find $\sqrt{20 - 21i}$ (where $i = \sqrt{-1}$) in the form $x + iy$ where x and y are real numbers.

(c) Consider an arbitrary complex number z $(= x + iy)$ where $i = \sqrt{-1}$. For what values of x and y does $\dfrac{z + 3i}{z + 2} = \dfrac{2z + 3i}{z + 4}$ hold?

6. (a) Find the rational numbers a, b such that

$$f(x) = \frac{1}{(2x + 3)(2x + 5)} = \frac{a}{2x + 3} + \frac{b}{2x + 5}.$$

(b) Let x take the values $0, 1, 2, ..., n$. Compute $f(0) + f(1) + ... + f(n)$. Deduce that the area between the curve $y = f(x)$ and the positive x-axis is at most $\frac{1}{6}$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2002

MATHEMATICS

Second Paper

Time allowed: *Two* hours.
Full marks for *five* correct solutions.

DO NOT USE GRAPH PAPER

1. (a) Find the term independent of x in the binomial expansion of

$$\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9.$$

(b) From the expansion of $\left(1 + \frac{4x}{5}\right)^{12}$ find

(i) the value of the greatest coefficient;

(ii) the value of the greatest term when $x = \frac{2}{3}$.

2. Let A, B, C be the three angles in a triangle and let a, b, c be the lengths of the sides opposite the respective angles A, B, C. Let r denote the radius of the circle inscribed in the triangle (and tangent to the sides of the triangle).

PROVE:

(a) $\Delta = sr$ where Δ is the area of the triangle and $s = \dfrac{a+b+c}{2}$, the so-called semi-perimeter of the triangle.

(b) $r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right) = a.$

(c) $r\cos\frac{A}{2} = a\left(\sin\frac{B}{2}\sin\frac{C}{2}\right).$

P.T.O.

3. (a) Solve the simultaneous equations:

$$\begin{aligned} 2x + 3y &= 2 \\ \frac{2}{x} - \frac{1}{y} &= 1 \end{aligned}$$

(b) If

$$\frac{ad}{b-c} = \frac{be}{c-a} = \frac{cf}{a-b}$$

for arbitrary numbers a, b, c, d, e, f with $a \neq b,\ b \neq c,\ c \neq a$, prove that $ad + be + cf = 0$.

(c) Consider the sequence a_n, where $a_n = \dfrac{1-(k+1)n}{2+kn}$, and k is a constant.
(i) Find the limit of the sequence when $k = 2$.
(ii) Find k when the limit of the sequence is 2.

4. (a) Show that $y = Ae^x + Be^{2x}$, where A and B are constants, is a solution of the equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

Given that $y = 1$ when $x = 0$ and $\dfrac{dy}{dx} = 0$ when $x = 0$, find A and B.

(b) Sketch the graph of

$$y = \frac{2x}{x+1}$$

for $x \geq 0$ and find the equation of the tangent to the curve at $x = 1$.

5. (a) Find the dimensions of the largest (in area) rectangle the perimeter of which has length 200cm.

(b) Find the volume of the solid generated by revolving the curve $y = x^2 + 1$ about the x–axis for $0 \leq x \leq 2$.

6. (a) Evaluate the following integrals:

$$\int x \cos x \, dx; \qquad \int_0^1 x^3\sqrt{x^4+1}\, dx; \qquad \int_0^{\pi/2} \cos^3 x \, dx.$$

(b) Sketch the graphs of the functions $y = x^3$ and $y = 3x - 2$ (do NOT use graph paper) and find the area between their graphs.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2015

MATHEMATICS

First Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

Log Tables: REQUIRED

1. (a) The coordinates of three vertices of a parallelogram $ABCD$ are:

$$A\,(-5,\,1),\;\; B\,(2,\,4),\; C\,(1,\,1).$$

calculate the area of the parallelogram

(b) A line divides a circle of radius length 2 into two unequal arcs. Given that the length of the smaller arc is $\frac{\pi}{3}$, determine the area of the region bounded by the smaller arc and the line.

(c) Determine all the values of θ, where $0 < \theta \leq 2\pi$, such that $\sin^2\theta = 1 - \cos\theta$.

2. (a) For $x > \dfrac{1}{2}$ solve the equation:

$$\log_5 x = 1 + \log_5\left(\frac{3}{2x-1}\right).$$

(b) Find the sum of all the natural numbers less than 2015 which are divisible by 7.

(c) The volume of a rectangular solid is 8cm^3, and its total surface area is 32cm^2. The three dimensions (length, width and height) are in geometric progression (a, ar, ar^2). Determine the sum of the length of all the 12 edges of the solid (in cm).

p.t.o.

3. (a) Find the equation of the circle passing through the points $(5,5)$, $(4,8)$, $(-7,-3)$, the co–ordinates of its centre and the length of the radius.

(b) Find the equation of the circle which touches the y–axis at the point (0, 3) and passes through the point (1, 0). Hence find the co–ordinates of its centre and the length of its radius.

(c) Find the radius and centre of the two circles in the first quadrant touching both the x–axis and the y–axis and the line $3x + 4y = 4$.

4. (a) If $(x,\, y)$ is a solution to the system of equations (i), (ii) below

$$\begin{aligned} xy &= 8 \qquad (i) \\ x^2y + xy^2 + x + y &= 63 \qquad (ii) \end{aligned}$$

determine the value of $x^2 + y^2$.

(b) There are four points on a line. There are four other points, none of which is on the line and no three of which are collinear. What is the greatest number of triangles that can be formed using three of these eight points as vertices?

5. (a) Writing $\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$ in modulus-argument form (or otherwise),

determine $\left(\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}\right)^{11}$ (where $i = \sqrt{-1}$).

(b) Determine the complex number z such that $\dfrac{2z - 3i}{z + 2} = -5 + i$ (where $i = \sqrt{-1}$).

(c) Show that the complex numbers z $(= x + iy)$ which satisfy

$$|z - 1 - i| = 2|z + 1 - 2i|$$

lie on a circle. Find the centre and radius of this circle.

6. (a) Consider the function $f(x) := \ln(1 + x^2)$. Calculate the first derivative $f'(x)$, also written $\dfrac{df}{dx}$. Calculate the second derivative $f''(x)$, also written $\dfrac{d^2f}{dx^2}$.
Hence or otherwise show that $f(x)$ has a minimum at $x = 0$ and two points of inflection at $x = \pm 1$.

(b) Find the rational numbers a, b such that

$$f(x) = \frac{1}{(x+5)(x+7)} = \frac{a}{x+5} + \frac{b}{x+7}.$$

Let x take the values $0, 1, 2, ..., n$.
Compute $f(0) + f(1) + ... + f(n)$. Deduce that the area between the curve $y = f(x)$ and the positive x-axis is at most $\dfrac{11}{60}$.

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FACULTY OF ENGINEERING

ENTRANCE EXAMINATION 2015

MATHEMATICS

Second Paper

Time allowed: *Two* hours
Engineering students take **5** questions out of 6
Computer Science and IT and Project and Construction Management students take **4** questions out of 6.

Log Tables: REQUIRED

1. (a) Find the equations of the lines represented by the equation

$$2x^2 + 5xy - 3y^2 + 7x + 14y + 5 = 0.$$

(b) A line containing the point P with coordinates $(5, 6)$ touches the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ at the point Q. Calculate the distance PQ.

(c) The lines $L_1 : 3x + 4y + 4 = 0$ and $L_2 : 5x + 12y + 28 = 0$ cut the x-axis at points A and B respectively. Let K be a point in the segment $[AB]$ which is equidistant from L_1 and L_2. Find (i) the coordinates of K and (ii) this distance from L_1 and L_2.

2. Let A, B, C be the three angles in a triangle and let a, b, c be the lengths of the sides opposite the respective angles A, B, C. Let r denote the radius of the circle inscribed in the triangle (and tangent to the sides of the triangle).
PROVE:

(a) $\Delta = sr$ where Δ is the area of the triangle and $s = \dfrac{a+b+c}{2}$, the so–called semi–perimeter of the triangle.

(b) $r\left(\cot\dfrac{B}{2} + \cot\dfrac{C}{2}\right) = a.$

(c) $r\cos\dfrac{A}{2} = a\left(\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right).$

p.t.o.

3. (a) How many different 15-letter "words" can be formed from the letters of the word **UNPREPOSSESSING**? Explain your answer. [Note that interchanging any of the four letters S among themselves does not give a different "word".]
How many of these "words" have all four **S**'s together?

(b) If repetitions are **NOT** allowed

(i) how many 3–digit numbers can be formed from the set $\{3, 4, 5, 6, 7, 8, 9\}$?

(ii) How many of these are **ODD** numbers?

(iii) How many are greater than 600?

(iv) How many are divisible by 4?

4. (a) Show that $y = Ae^x + Be^{2x}$, where A and B are constants, is a solution of the equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

Given that $y = 1$ when $x = 0$, and $\frac{dy}{dx} = 0$ when $x = 0$, determine the value of the constants A and B.

(b) Consider the curve

$$y = \frac{2x}{x+1}$$

for $x \geq 0$. By differentiating y with respect to x, or otherwise, find the equation of the tangent to the curve at $x = 1$.

5. (a) Differentiate with respect to x:

(i) $(\sin x)^2$ (ii) $\sqrt{\frac{x+1}{x+2}}$; (iii) $2x \ln x$.

(b) Find the maxima and minima of the function

$$f(x) = x^3 - x^2 - x + 2.$$

and sketch its graph on the interval $(-2, 2)$ (do **NOT** use graph paper). How many real roots does the function $f(x)$ have?

6. (a) Evaluate the following integrals:

$$\int \sin 4x \; dx; \qquad \int_0^1 2x\sqrt{x^2+1} \; dx; \qquad \int_0^{\pi/2} \sin^3 x \; dx.$$

(b) Sketch the graphs of the functions $y = x^2$ and $y = 2 - x$ (do **NOT** use graph paper) and find the area between their graphs.